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1 SEM TDC MTH M 1

2015

(November)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 32 (Backlog) / 24 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the domain of all sequences. 1

(b) Choose the correct answer for the following : 1

A sequence $\{f_n\}$ is said to be bounded below if there exists a real number k such that—

(i) $f_n \geq k,\ n \in N$ *(ii)* $f_n \leq k,\ n \in N$

(iii) $f_n = k,\ n \in N$ *(iv)* $\frac{1}{f_n} \geq k,\ n \in N$

(c) Write the conditions under which a sequence diverges to ∞ and $-\infty$. 2

(d) Show that $\{f_n\}$, where $f_n = \frac{(-1)^n}{n}$ is a Cauchy sequence. 2

(e) Show that the set of all limit points of a bounded sequence is bounded. 4

Or

Show that every bounded sequence with a unique limit point is convergent.

2. (a) Write the condition under which the infinite series Σu_n diverges. 1

(b) If the series $\sum_{n=1}^{\infty} u_n$ converges, then write the value of $\lim_{n \to \infty} u_n$. 1

(c) Test for the convergence of the series

$$\sum_{n=1}^{\infty} \cos \frac{1}{n^2}$$ 2

(d) Write the statement of Raabe's test. 2

(e) Show that the infinite series $\sum \frac{1}{n^2 + a^2}$ is convergent. 4

Or

Test the convergence of the series

$$\sum \frac{n!}{n^n}$$

(f) State Cauchy's condition of convergence for an infinite series and using this principle, test the convergence of the series $\sum \frac{1}{n}$. 2+3

3. *(a)* Write the transformed equation of $8x^3 - 9x^2 + 1 = 0$ to solve by Cardan's method. 1

(b) Choose the correct answer for the following : 1

Every equation of odd degree

(i) has no real root

(ii) has at least two real roots

(iii) has only complex roots

(iv) has at least one real root

(c) Write the nature of the roots of the equation $x^3 + px + q = 0$, if p and q are positive. 2

(d) If $2+3i$ is a root of the equation $x^3-6x^2+21x-26=0$, then find the value of the real root. 2

(e) Solve the following by Cardan's method : 4

$$x^3-6x-9=0$$

Or

Find the equation whose roots are the squares of the roots of the equation $2x^3-3x^2+4x-5=0$.

(f) Find the sum of the fourth powers of the roots of $x^3-x-1=0$. 3

Or

If α, β, γ are the roots of the equation $x^3+qx+r=0$, then find the equation whose roots are $\alpha-\frac{1}{\beta\gamma}$, $\beta-\frac{1}{\alpha\gamma}$, $\gamma-\frac{1}{\alpha\beta}$.

(g) If α, β, γ are the roots of the equation $6x^3-11x^2-3x+2=0$ and are in harmonic progression, then find the value of $\gamma\alpha$. 2

GROUP—B

(Trigonometry)

4. *(a)* Write the value of

$$\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}$$ 1

(b) Expand $\sin^2 x$ in powers of x. 2

(c) If n is odd, then write the last term in the expansion of $\sin n\theta$. 1

(d) If α and β are the roots of the equation $x^2-2x+4=0$, then find the value of $\alpha^n+\beta^n$. 4

Or

Prove that

$$\cos 5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$$

5. *(a)* If z is a complex number, then write the period of e^z. 1

(b) Prove that

$$\tan^{-1}(e^{i\theta})=\frac{\pi}{4}+\frac{1}{2}i\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$ 4

Or

Prove that

$$\sin(\log i^i)=-1$$

(Turn Over)

6\. *(a)* If x is complex and $\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \ldots$, then write the interval in which the real part of $\tan^{-1} x$ lies. 1

(b) Evaluate the value of

$$\frac{2}{3} + \frac{1}{7} - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) - \cdots$$ 3

Or

Expand $\tan^{-1}\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)$ as a power series in $\tan\theta$.

7\. *(a)* Show that $\text{sech}^2 x = 1 - \tanh^2 x$. 2

(b) Separate into real and imaginary parts of $\sin(\alpha + i\beta)$. 2

(c) Find the sum of the series

$$\sin\alpha + n\sin(\alpha + \beta) + \frac{n(n-1)}{2!}\sin(\alpha + 2\beta) + \ldots \text{ to } (n+1) \text{ terms}$$ 4

Or

Find the sum of the series

$$\cos\theta + \sin\theta + \frac{\cos^2\theta \sin 2\theta}{2!} + \frac{\cos^3\theta \sin 3\theta}{3!} + \ldots$$

GROUP—C

(Vector Calculus)

8. *(a)* If $\vec{r} = (x^2y^2 - x^3)\hat{i} + e^{xy}\hat{j} + x^2 \sin y\hat{k}$, then find $\frac{\partial \vec{r}}{\partial x}$. 2

(b) Show that $\vec{v} = z\hat{i} + x\hat{j} + y\hat{k}$ is a solenoidal vector. 1

(c) Find $\nabla\phi$, at $(1, -2, 2)$, where

$$\phi = x^2z - y^2z + x^3y$$ 2

(d) Find $\nabla\left(\frac{1}{r}\right)$, where

$$r = |\vec{r}|,\ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$ 2

(e) Show that div curl $\vec{A} = 0$. 3

Or

Evaluate $\nabla \cdot (r^n \vec{r})$.

(f) Show that

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$ 5

Or

Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in a direction towards a point $(2, -3, 6)$.

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