5 SEM TDC STS M 1 (N/O)

2019

(November)

STATISTICS

(Major)

Course : 501

(Estimation)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

1. Select the correct one out of the given alternatives : $1 \times 8 = 8$

(a) Estimation is possible only in case of a

(i) parameter

(ii) sample

(iii) random sample

(iv) None of the above

(b) $\frac{1}{n}\sum_{i} x_i$ for $i = 1, 2, ..., n$ is called

(i) estimation

(ii) estimate

(iii) estimator

(iv) interval estimate

(c) The sample variance is not a/an _____ estimator, but it is a/an _____ estimator for population.

(i) unbiased, consistent respectively

(ii) biased, efficient respectively

(iii) consistent, unbiased respectively

(iv) None of the above

(d) If the variance of an estimator attains its Cramer-Rao lower bound for variance, then the estimator is

(i) most efficient

(ii) sufficient

(iii) unbiased

(iv) All of the above

(e) If a sufficient statistic exists for a parameter, then it will be a function of

(i) moment estimator

(ii) ML estimator

(iii) unbiased estimator

(iv) None of the above

(f) The method of moments was invented by

(i) Neyman

(ii) Fisher

(iii) Karl Pearson

(iv) Snedecor

(g) By decreasing the sample, the confidence interval becomes

(i) narrower

(ii) wider

(iii) fixed

(iv) None of the above

(h) A range of values within which the population parameter is expected to occur is called

(i) confidence co-efficient

(ii) confidence interval

(iii) confidence limits

(iv) level of significance

2. Answer the following in brief : 2×8=16

(a) Differentiate between point estimation and interval estimation.

(b) $x_1, x_2, \ldots, x_n$ is a random sample from a normal population, $N(\mu, 1)$. Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

(c) State the sufficient condition for consistency.

(d) What do you mean by minimum variance unbiased estimator (MVUE)?

(e) State the invariance property of maximum likelihood estimator (MLE).

(f) What do you mean by likelihood function?

(g) Find the 95% confidence limit and confidence interval for population mean, μ of normal distribution.

(h) Explain confidence limits.

3. *(a)* *(i)* Let X_1, X_2, X_3 and X_4 be independent random variables such that $E(X_i = \mu)$ and $V(X_i = \sigma^2)$ for $i = 1, 2, 3, 4$. If

$$Y = \frac{X_1 + X_2 + X_3 + X_4}{4},\ Z = \frac{X_1 + X_2 + X_3 + X_4}{5}$$

$$T = \frac{X_1 + 2X_2 + X_3 - X_4}{4}$$

examine whether Y, Z and T are unbiased estimators of μ. What is the efficiency of Y relative to Z? 5

(ii) If $X_1, X_2, ..., X_n$ are random observations on a Bernoulli variate X, taking the value 1 with probability p and the value 0 with probability $(1 - p)$, then show that

$$\frac{\Sigma x_i}{n}\left(1 - \frac{\Sigma x_i}{n}\right)$$

is a consistent estimator of $p(1 - p)$. 5

Or

(b) *(i)* What is the necessary and sufficient condition for T to be sufficient estimator for θ? Let $x_1, x_2, ..., x_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for θ. 5

(ii) If T_1 and T_2 are two unbiased estimators $\gamma(\theta)$ having the same variance and ρ is the correlation coefficient between them, then show that $\rho \geq 2e - 1$, where e is the efficiency of the each estimator. 5

4. *(a)* *(i)* Obtain the minimum variance bound estimator (MVBE) for μ of the normal population $N(\mu, \sigma^2)$, where σ^2 is known. 5

(ii) If T_1 and T_2 are two unbiased estimators of a parameter θ with variances ${\sigma_1}^2$ and ${\sigma_2}^2$ and correlation co-efficient ρ, then obtain the best linear combination of T_1 and T_2. 5

Or

(b) *(i)* Show that

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

in random sampling from

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad ; \quad 0 < x < \infty$$
$$= 0 \quad ; \quad \text{otherwise}$$

is a minimum variance bound (MVB) estimator of θ and has variance $\frac{\theta^2}{n}$. 5

(ii) Write a short note on Rao-Blackwell theorem. 5

5. Discuss optimal properties of maximum likelihood method of estimation. 5

6. (a) (i) Find the MLE for the parameter λ of the Poisson distribution on the basis of a sample of size n. Also find its variance. 5

(ii) Prove that if a sufficient estimator exists, it is a function of MLE. 5

Or

(b) (i) Explain the method of moments in the theory of estimation. 5

(ii) Discuss the method of least squares in the theory of estimation. 5

7. (a) In random sampling from normal population $N(\mu, \sigma^2)$, find the MLE for—

(i) μ, when σ^2 is known;

(ii) σ^2, when μ is known;

(iii) the simultaneous estimation of μ and σ^2. 8

Or

(b) Let $X_1, X_2, ..., X_n$ be a random sample from the p.d.f

$$f(x, \theta) = \theta e^{-\theta x} \quad ; \quad 0 < x < \infty, \quad \theta > 0$$
$$= 0 \quad ; \quad \text{otherwise}$$

Estimate θ using the method of moment. Describe some characteristics of the method of moment. 6+2=8

8. Write an explanatory note on confidence interval and confidence co-efficient. 5

9. (a) For the distribution

$$f(x, \theta) = \theta e^{-\theta x} \quad ; \quad 0 < x < \infty$$

obtain $100(1-\alpha)\%$ confidence interval for θ for large samples. 8

Or

(b) Explain how confidence interval can be constructed for large samples. A random sample of size 100 has mean 15, the population variance 25. Find the interval estimate of population mean with confidence levels of 95% and 99%.

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) The theory of estimation was founded by

(i) Laplace

(ii) Fermat

(iii) Fisher

(iv) None of them

(b) Which of the following statistics is unbiased estimator?

(i) The sample mean

(ii) The sample variance

(iii) The sample proportion

(iv) All of the above

(c) If $t_n \xrightarrow{p} \theta$, than t_n is a/an ____ estimator of θ.

(i) unbiased

(ii) consistent

(iii) efficient

(iv) sufficient

(d) Factorization theorem for sufficiency is also known as

(i) Fisher-Neyman theorem

(ii) Cramer-Rao theorem

(iii) Rao-Blackwell theorem

(iv) None of the above

(e) MLE of θ, in a random sample of size n from $U(0, \theta)$ is

(i) the sample mean

(ii) the sample median

(iii) the largest-order statistics

(iv) the smallest-order statistics

(f) In common, the estimators obtained by the method of MLE, are

(i) more efficient

(ii) less efficient

(iii) Cannot say about efficiency

(iv) None of the above

(g) If $(1-\alpha)$ is increased, the width of a confidence interval is

(i) decreased

(ii) increased

(iii) consistent

(iv) same

(h) A 95% confidence interval for the mean of a population is such that

(i) it contains 95% of the values in the population

(ii) there is a 95% chance that it contains all the values in the population

(iii) there is a 95% chance that it contains the mean of the population

(iv) there is a 95% chance that it contains the standard deviation of the population

2. Answer the following in brief : 2×8=16

(a) What are estimate, estimator and estimation?

(b) What are the important criteria of a good estimator?

(c) Define consistent estimator.

(d) State the Cramer-Rao inequality.

(e) State the invariance property of MLE.

(f) Mention the names of various methods of estimation of a parameter.

(g) Define confidence interval and confidence co-efficient.

(h) What do you mean by 99% confidence limits and confidence intervals for the estimation of population mean?

3. (a) (i) If $x_1, x_2, \ldots, x_n$ is a random sample where variate x taking the value 1 with probability p and taking the value 0 with probability $1-p$, then show that $\bar{x}(1-x)$ is a consistent estimator of $p(1-p)$. 5

(ii) For a random sample $x_1, x_2, \ldots, x_n$ taken from a finite population, show that

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

is not an unbiased estimator of the parameter σ^2, but

$$\frac{n}{n-1}s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

is unbiased. 3+2=5

Or

(b) (i) Let X_1, X_2, X_3 and X_4 be independent random variables such that $E(x_i) = \mu$ and $V(x_i) = \sigma^2$ for $i = 1, 2, 3, 4$. If

$$Y = \frac{X_1 + X_2 + X_3 + X_4}{4}, \; Z = \frac{X_1 + X_2 + X_3 + X_4}{5}$$

$$T = \frac{X_1 + 2X_2 + X_3 - X_4}{4}$$

examine whether Y, Z and T are unbiased estimators of μ. What is the efficiency of Y relative to Z? 5

(ii) Let $x_1, x_2, ..., x_n$ be a random sample of n observations from the population having p.d.f.

$$f(x) = \theta x^{\theta - 1} \; ; \; 0 \le x \le 1, \; \theta > 0$$

Find the sufficient statistic for θ. 5

4. (a) (i) Explain minimum variance unbiased estimator (MVUE) with examples. If t_n is an unbiased estimator of θ, find Cramer-Rao bound of it. 5

(ii) State the sufficient condition for consistency. Give the statements of the factorization theorem and Rao-Blackwell theorem. 1+4=5

Or

(b) (i) If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$ having the same variance and ρ is the correlation coefficient between them, then show that $\rho \geq 2e - 1$, where e is the efficiency of each estimator. 5

(ii) Let $x_1, x_2, \ldots, x_n$ be a random sample from an $N(\mu, \sigma^2)$ population. Find the sufficient estimators for μ and σ^2. 5

5. State the important properties of MLE. 5

6. (a) (i) State and explain the principle of MLE. 5

(ii) Find the MLE for the parameter λ of the Poisson distribution on the basis of a sample of size n. Also find its variance. 5

Or

(b) (i) Explain the method of minimum chi square in the theory of estimation. 5

(ii) Let $x_1, x_2, \ldots, x_n$ be a random sample of n observations from a Poisson population whose p.d.f. is

$$p(x) = \frac{e^{-\theta}\theta^x}{x!}; \qquad x = 0, 1, 2, \ldots$$

Find the estimation of θ by the method of moment. 5

7. (a) Explain the method of least squares in the theory of estimation and write the assumptions for estimation. 4+4=8

Or

(b) In random sampling from normal population $N(\mu, \sigma^2)$, find the MLE for—

(i) μ, when σ^2 is known;

(ii) σ^2, when μ is known;

(iii) the simultaneous estimation of μ and σ^2. 8

8. Distinguish between point estimation and interval estimation. 5

9. (a) Obtain $100(1-\alpha)\%$ confidence interval for the parameter θ and σ^2 of the normal distribution

$$f(x\,;\,\theta, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2}$$

$$-\infty < \theta < \infty, \quad \sigma > 0, \quad -\infty < x < \infty$$ 8

(Turn Over)

Or

(b) For the distribution

$$f(x, \theta) = \theta e^{-\theta x}, \ \ 0 < x < \infty$$

obtain the $100(1-\alpha)\%$ confidence interval for θ (for large sample).

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