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5 SEM TDC STS M 2 (N/O)

2019

(November)

STATISTICS

(Major)

Course : 502

(Testing of Hypothesis)

Time : 3 hours

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 80

Pass Marks : 24

1. Fill in the blanks : 1×8=8

(a) The region of rejection of H_0, when H_0 is true is called _____.

(b) Failing to reject the null hypothesis when it is false is defined as a _____.

(c) If the *p*-value is _____ then level of significance, that would lead to rejection of the null hypothesis H_0 with the given data.

(d) The χ^2-test for ____ is tested for hypothesis related to categorical data.

(e) One of the important assumptions for Student's t-test is that the population ____ is unknown.

(f) ____ tests are based on order statistic theory.

(g) Non-parametric tests are not defined for ____ parameters.

(h) Most of the non-parametric methods utilize measurements on ____ scale.

2. Answer the following questions : $2 \times 8 = 16$

(a) What are simple and composite statistical hypotheses?

(b) Define MP critical region and UMP critical region.

(c) State Neyman-Pearson lemma.

(d) What is the F-test for testing the significance of the linearity of regression?

(e) Explain statistic and parameter.

(f) Explain how the run test can be used to test randomness.

(g) Why are the non-parametric methods referred to as 'distribution-free' methods?

(h) What are the advantages of non-parametric tests?

3. (a) What is statistical hypothesis? Define two types of errors and power of a test with reference to testing of hypothesis. Explain how the best critical region is determined. State clearly the theorem which is used to determine the best critical region for simple hypothesis at a given significance level. 2+4+3+3=12

Or

(b) A textile fibre manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0·5 kilogram. The company wishes to test the hypothesis $H_0: \mu = 12$ against $H_1: \mu_1 < 12$, using a random sample of four specimens.

(i) What is the type-I error probability if the critical region is defined as $\bar{x} < 11 \cdot 5$ kilograms?

(ii) Find β for the case where the true mean elongation is 11·25 kilograms.

(iii) Find β for the case where the true mean is 11·5 kilograms. 4+4+4=12

4. (a) What do you mean by unbiased test and unbiased critical region? Prove that—

(i) if W be an MPCR of size α for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, then it is necessarily unbiased;

(ii) if W be a UMPCR of size α for testing $H_0: \theta = \theta_0$ against $H_1: \theta \in \Theta_1$, then it is also unbiased. 3+4+2=9

Or

(b) Define 'likelihood ratio test'. Discuss the general method of construction of likelihood ratio test. Mention some properties of likelihood ratio test. 3+3+3=9

5. Answer any *two* of the following questions : 9×2=18

(a) What is the procedure for testing the significance of statistical hypothesis for large sample? Describe the method of large sample test of significance for single proportion.

(b) Consider the following 2×2 table of observed frequencies based on random samples (with replacement) of sizes $n_{.1}$ and $n_{.2}$ from two populations :

		Population I	Population II	Total
Class A	:	n_{11}	n_{12}	$n_{1.}$
Class B	:	n_{21}	n_{22}	$n_{2.}$
Total	:	$n_{.1}$	$n_{.2}$	n

(i) Define the χ^2-statistic to be used for the test of homogeneity of the two populations.

(ii) Show that,

$$\chi^2 = \frac{n(n_{11}n_{22} - n_{12}n_{21})^2}{(n_{1.}n_{.1}n_{2.}n_{.2})}$$

(c) It is desired to test the hypothesis that the mean of a normal population is $\mu = \mu_0$ against the alternative that $\mu \neq \mu_0$. Explaining the assumptions involved, develop the statistic suitable for testing this hypothesis if the size of the sample is small. What modification do you suggest when the sample size is large?

(d) Discuss the application of F-test in testing if the two variances are homogeneous. Explain why the larger variance is placed in the numerator of F-statistic.

6. *(a)* Under what circumstances normally sign test is conducted? Describe the sign test. 3+7=10

Or

(b) Stating the underlying assumptions and the null hypothesis develop the median test. 1+2+7=10

7. *(a)* The win-lose record of a certain basketball team for its last 50 consecutive games was as follows :

W W W W W W L W W W W W W L
W L W W W L L W W W W L W W W
L L W W W W W W L L W W L L L W
W L W W W

Apply run test to test the sequence of wins and loses is random. 7

Or

(b) Describe briefly Kolmogorov-Smirnov test for goodness of fit. 7

(Old Course)

Full Marks : 80

Pass Marks : 32

1. Fill in the blanks : 1×8=8

(a) The theory of testing of hypothesis was propounded by ____ in 1933.

(b) The probability of type–I error is given by ____.

(c) Critical region provides the criterion for ____ of null hypothesis.

(d) From the normal probability table, we know that $P(|Z| \leq \_\_\_\_) = 0{\cdot}9973$.

(e) The significance of an observed partial correlation coefficient can be tested by ____ test.

(f) SPRT was developed by ____.

(g) In SPRT, the sample size is a ____ variable.

(h) The number of elements in a run is usually called the ____ of the run.

(*Turn Over*)

2. Answer the following in brief : 4×4=16

(a) Discuss the general method of construction of likelihood ratio test.

(b) Describe the χ^2-test of independence of attributes.

(c) Explain how the sequential test procedure differs from the Neyman-Pearson test procedure.

(d) Distinguish between sign test and Wilcoxon signed rank test.

3. (a) Explain any *two* of the following terms : 5×2=10

(i) Errors of first kind and second kind

(ii) The best critical region

(iii) Power function of a test

(iv) Level of significance

Or

(b) State and prove Neyman-Pearson lemma for testing a simple null hypothesis against a simple alternative. 10

4. (a) Define MP region and UMP region. Show that an MP region is necessarily unbiased. 8

Or

(b) Show that the likelihood ratio principle leads to the same test, when testing a simple hypothesis against an alternative simple hypothesis, as that given by Neyman-Pearson theorem. 8

5. Answer any *two* of the following questions : $7\times2=14$

(a) What are the various steps involved in testing of significance of a statistical hypothesis? Describe the test of significance for large samples of attributes for single proportion.

(b) What are the assumptions made in the Student's t-test? Mention some applications of Student's t as the test of significance. Explain the t-test for testing the significance of the difference between two sample means.

(c) Discuss the Chi-square test of goodness of fit of a theoretical distribution to an observed frequency distribution. How are the degrees of freedom ascertained when some parameters of the theoretical distribution have to be estimated from the data?

(d) Describe the F-test for equality of two population variances. Name the technique of which F-test for equality of several means is carried out.

6. Answer any *two* of the following questions : $7 \times 2 = 14$

(a) Describe Wald's sequential probability ratio test.

(b) Define OC function and ASN function on sequential analysis. Derive their approximate expressions for the SPRT of a simple hypothesis against a simple alternative.

(c) Let X has the distribution, $f(x, \theta) = \theta^{x}(1-\theta)^{1-x}$; $x = 0, 1$; $0 < \theta < 1$. For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, construct SPRT.

7. (a) Explain what is meant by non-parametric methods. How do they differ from parametric methods? Derive the sign test, stating clearly the assumptions made. 2+3+5=10

Or

(b) Describe the procedure of median test when there are two independent samples. What NP test would you use when the two samples are related? 8+2=10

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