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3 SEM TDC STS M 1 (N/O)

2019

(November)

STATISTICS

(Major)

Course : 301

(Probability and Distribution—I)

Full Marks : 80

Pass Marks : 24/32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the given alternatives : 1×8=8

(a) For the sample space $S = \{e_1, e_2, e_3\}$

(i) $P(e_1) + P(e_2) + P(e_3) > 1$

(ii) $P(e_1) + P(e_2) + P(e_3) = 1$

(iii) $P(e_1) + P(e_2) + P(e_3) < 1$

(iv) None of the above

(b) If A and B are two independent events, then $P(\overline{A} \cap \overline{B})$ is equal to

(i) $P(\overline{A}) \cdot P(\overline{B})$

(ii) $1 - P(A \cup B)$

(iii) $[1 - P(A)][1 - P(B)]$

(iv) All of the above

(c) If A_1, A_2 and A_3 are three mutually exclusive events, then the probability of their union is equal to

(i) $P(A_1)P(A_2)P(A_3)$

(ii) $P(A_1)+P(A_2)+P(A_3)-P(A_1A_2A_3)$

(iii) $P(A_1)+P(A_2)+P(A_3)$

(iv) $P(A_1)P(A_2)+P(A_1)P(A_3)+P(A_2)P(A_3)$

(d) If $F(x)$ is a distribution function of a random variable, then

(i) $F(x)>1$

(ii) $F(x)<0$

(iii) $0 \leq F(x) \leq 1$

(iv) None of the above

(e) For two random variables X and Y with $E(X)=2$ and $E(Y)=4$, $E(2X-5Y)$ will be

(i) -16

(ii) 24

(iii) -2

(iv) 108

(f) If X is a random variable which can take only non-negative values, then

(i) $E(X^2)=[E(X)]^2$

(ii) $E(X^2) \geq [E(X)]^2$

(iii) $E(X^2) \leq [E(X)]^2$

(iv) None of the above

(g) If X and Y are two variables, then there can be at most

(i) one regression line

(ii) three regression lines

(iii) two regression lines

(iv) four regression lines

(h) The conditional probability density function of X given $Y = y$ for a joint density $f_{XY}(x, y)$ can be found by the formula

(i) $f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_X(x)}$

(ii) $f_{X|Y}(x|y) = f_{Y|X}(y) f_{XY}(x, y)$

(iii) $f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)}$

(iv) None of the above

2. Answer the following questions : 2×8=16

(a) Write down the axiomatic definition of probability.

(b) State and prove the addition theorem of probability of two events A and B.

(c) A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$. Find (i) $P(B)$ and (ii) $P(A \cap \overline{B})$.

(d) What are the properties of distribution function?

(e) Prove that if X and Y are two independent random variables, then $E(XY) = E(X)E(Y)$.

(f) Write the properties of moment generating function.

(g) Find the correlation coefficient between random variables X and Y, if $V(X) = V(Y) = \frac{1}{4}$ and $V(X - Y) = \frac{1}{3}$.

(h) Define joint probability mass function, marginal probability mass function and conditional probability mass function.

3. What are meant by mutually exclusive and exhaustive events associated with random experiments? Find $P(A)$, given that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$, where A, B and C are mutually exclusive and exhaustive events. 4

4. (a) If A, B and C are random events in a sample space and if A, B and C are pairwise independent and A is independent of $(B \cup C)$, then prove that A, B and C are mutually independent. 4

(b) For any two events A and B, prove that $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$. 3

5. Answer any *two* of the following questions : 5×2=10

(a) Two dice are thrown once. Find the probability of getting an odd number on the first die or total of 7.

(b) The probabilities of X, Y and Z becoming manager are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z become manager are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively. If bonus scheme has been introduced, what is the probability that the manager appointed was X?

(c) State and prove Bayes' theorem.

6. Answer any *three* of the following questions : 7×3=21

(a) Let X be a continuous random variable with probability density function given by—

$$f(x) = \begin{cases} kx & , \ 0 \le x < 1 \\ k & , \ 1 \le x < 2 \\ -kx + 3k, & 2 \le x < 3 \\ 0 & , \ \text{elsewhere} \end{cases}$$

(i) Determine the constant k.

(ii) Also determine $F(x)$, the distribution function.

(b) Define mathematical expectation of discrete and continuous random variable and function. Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.

(c) Let the random variable X assumes the value r with the probability law

$$P(X = r) = q^{r-1}p;\ r = 1, 2, 3\cdots$$

Find the moment generating function of X and hence its mean and variance.

(d) For cumulant generating function $k_X(t) = \log_e M_X(t)$, show that

$$\text{mean} = k_1$$
$$\mu_2 = k_2 = \text{variance}$$
$$\mu_3 = k_3$$
$$\mu_4 = k_4 + 3k_2^2$$

(e) (i) Find the characteristic function, if $P(x) = \dfrac{e^{-\lambda}\lambda^x}{\underline{|x}};\ x = 0, 1, \cdots\infty$

(ii) A random variable X assumes the values $\lambda_1, \lambda_2, \cdots$ with probabilities $U_1, U_2, \cdots$ respectively. Show that $P_k = \dfrac{1}{\underline{|k}}\sum_{j=0}^{\infty} U_j e^{-\lambda_j}(\lambda_j)^k;\quad \lambda_j > 0,$

$\Sigma U_j = 1$ is a probability distribution. Find its generating function.

7. Answer any *two* of the following questions : 5×2=10

(a) Two fair dice are thrown simultaneously. Let X denotes the number on the first die and Y denotes the number on the second die. Find the following probabilities :

(i) $P(X+Y=8)$

(ii) $P[(X+Y)\geq 8]$

(iii) $P(X+Y=6|Y=4)$

(b) A two-dimensional random variable (X, Y) has a joint probability mass function $P(x, y)=\frac{1}{27}(2x+y)$, where x and y can assume only the integer values 0, 1 and 2. Find the marginal distribution of X and Y and also the conditional distribution of Y for $X=x$.

(c) Suppose that two-dimensional continuous random variable (X, Y) has joint probability density function given by

$$f(x, y)=\begin{cases}6x^2y, & 0<x<1,\ 0<y<1\\ 0 & , \text{elsewhere}\end{cases}$$

(i) Verify that $\int_0^1\int_0^1 f(x, y)\,dxdy=1$.

(ii) Find $P\left(0<X<\frac{3}{4}, \frac{1}{3}<Y<2\right)$, $P(X+Y<1)$, $P(X>Y)$ and $P(X<1|Y<2)$.

8. What do you mean by regression? Define regression coefficient. 4

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