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1 SEM TDC STSH (CBCS) C 2

2019

(December)

STATISTICS

(Core)

Paper : C–2

(Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) $\lim_{x \to 0} \frac{\sin x}{x}$ is

(i) 0

(ii) 1

(iii) ∞

(iv) None of the above

(b) A function $f(x)$ is said to be continuous at $x = a$, if

(i) $\lim_{x \to a} f(x)$ exists

(ii) $f(a)$ exists

(iii) $\lim_{x \to a} f(x) = f(a)$

(iv) $\lim_{x \to a} f(x) \neq f(a)$

(c) If $z = f(x, y)$ be a homogeneous function of x, y of degree n, then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is

(i) $n(n-1)z$

(ii) nz

(iii) n

(iv) z

(d) The value of the integral $\int uv dx$ is

(i) $u\int v dx + \int\left[\frac{du}{dx}\int v dx\right]dx$

(ii) $u\int v dx - \int\left[\frac{du}{dx}\int v dx\right]dx$

(iii) $v\int u dx + \int\left[\frac{du}{dx}\int v dx\right]dx$

(iv) $v\int u dx - \int\left[\frac{du}{dx}\int v dx\right]dx$

(e) The value of $\int_1^2 \int_0^{3y} y\,dy\,dx$ is

(i) 3

(ii) 5

(iii) 7

(iv) 9

(f) $y = A\cos x + B\sin x$ is a solution of the differential equation

(i) $\frac{dy}{dx} = -A\sin x + B\cos x$

(ii) $\frac{d^2y}{dx^2} = -A\cos x - b\sin x$

(iii) $\frac{d^2y}{dx^2} + y = 0$

(iv) All of the above

(g) The differential equation $Mdx + Ndy = 0$ is exact if and only if

(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(ii) $\frac{\partial^2 M}{\partial y^2} = \frac{\partial^2 N}{\partial x^2}$

(iii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iv) $\frac{\partial^2 M}{\partial x^2} = \frac{\partial^2 N}{\partial y^2}$

(h) The equation of the form

$$\frac{dy}{dx} + Py = Qy^n$$

is called

(i) Clairaut's equation

(ii) Bernoullis' equation

(iii) Cauchy's equation

(iv) None of the above

2. Answer the following : $2 \times 8 = 16$

(a) Find :

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

(b) If $u = \frac{x}{x+y}$ and $v = x + y$, then find $J\left(\frac{x, y}{u, v}\right)$.

(c) Define singular point of inflexion of function.

(d) Define Beta function and Gamma function.

(e) Prove that

$$\int_{-a}^{+a} f(x)\,dx = 0 \text{ or } 2\int_0^a f(x)\,dx$$

according as $f(x)$ is an odd or an even function of x.

(f) Write the general form of the first-order and first-degree differential equation.

(g) Write the general form of total or single differential equation in three variables x, y, z.

(h) Find a partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$

3. (a) (i) Define continuity of a function at the end points and write the properties of a continuous function. 3+4=7

(ii) Show that the function defined by

$$f(x) = \begin{cases} (1+2x)^{1/x} & , \text{ for } x \neq 0 \\ e^2 & , \text{ for } x = 0 \end{cases}$$

is continuous at $x = 0$. 3

Or

(b) (i) If $u = e^{xyz}$, then show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2) \cdot e^{xyz}$$ 5

(ii) State and prove Euler's theorem of homogeneous function. 5

(Turn Over)

4. *(a)* *(i)* If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then find $\frac{d^2y}{dx^2}$. 5

(ii) State and prove the Leibnitz theorem for nth derivative of the product of two functions. 5

Or

(b) *(i)* A random sample of size 400 is to be collected from two villages A and B for a socio-economic survey. The cost of collecting m units from A and n units from B is given by the cost function

$$f(m, n) = 3m^2 + mn + 2n^2 + 250$$

Use the method of Lagrange's multiplier to determine m and n in such a way that the cost of collecting data is minimum. 5

(ii) Define Jacobian of function of two variables. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$

2+3=5

5. Answer any *three* of the following : 5×3=15

(*a*) Evaluate :

$$\int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

(*b*) Find :

$$\int_0^{\log x} \int_1^{\log y} e^{x+y} \, dy \, dx$$

(*c*) By using the transformation $x + y = u$, $y = uv$, show that

$$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} \, dx \, dy = \frac{1}{2}(e-1)$$

(*d*) Change the order of integration and evaluate the integral

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dx \, dy$$

(*e*) Derive the following relation between Beta and Gamma functions :

$$\beta\,(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$$

(*f*) Prove that

$$\int_0^{\frac{\pi}{2}} \sin^m \theta \cos^n \theta = \frac{\Gamma\frac{m+1}{2} \, \Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}$$

(Turn Over)

6. Answer any *three* of the following : 4×3=12

(a) What is exact differential equation? Write the necessary and sufficient condition for the ordinary differential equation to be exact.

(b) What is homogenous linear differential equation? Discuss the working rule for solving a homogeneous linear differential equation.

(c) Solve :

$$(x+y+1)\frac{dy}{dx}=1$$

(d) Solve :

$$(D^2-2D+1)\,y=x^2e^{3x}$$

(e) Solve :

$$(y+z)\,dx+dy+dz=0$$

(f) Write the general form of Clairaut's equation. How to solve such equations?

7. (a) What is Lagrange's equation of linear partial differential equation of order one? Discuss the working rule for solving the Lagrange's equation. Solve the following differential equation by Lagrange's method : 3+3+3=9

$$y^2p+xyq=x(z-2y)$$

Or

(b) What is non-linear partial differential equation? Write different forms of first order non-linear partial differential equation. Solve the following differential equation by Charpit's method : 2+3+4=9

$$z = px + qy + p^2 + q^2$$

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