5 SEM TDC STS M 1 (N/O)

2018

(November)

STATISTICS

(Major)

Course : 501

(Estimation)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) An estimator T_n is said to be sufficient statistic for a parameter $\psi(\theta)$, if it contains all the informations which are contained in the

(i) population

(ii) parameter $\gamma(\theta)$

(iii) sample

(iv) None of the above

(b) The estimator T is called best estimator of a parametric function $\psi(\theta)$, if its variance is

(i) greater than or equal to the variance of any other estimator of $\psi(\theta)$

(ii) less than or equal to the variance of any other estimator of $\psi(\theta)$

(iii) equal to the variance of any other estimator of $\psi(\theta)$

(iv) None of the above

(c) The estimators obtained by the method of moments as compared to ML estimators are

(i) less efficient

(ii) more efficient

(iii) equally efficient

(iv) None of the above

(d) Factorization theorem for sufficiency is known as

(i) Rao-Blackwell theorem

(ii) Cramer-Rao theorem

(iii) Fisher-Neyman theorem

(iv) None of them

(e) The maximum likelihood estimators which are obtained by maximizing the function of joint density of random variables are generally

(i) unbiased and consistent

(ii) unbiased and inconsistent

(iii) consistent and invariant

(iv) invariant and unbiased

(f) The estimators by method of moments are

(i) consistent and unbiased

(ii) efficient and sufficient

(iii) inconsistent but unbiased

(iv) None of the above

(g) The 95% confidence limits for unknown parameter μ of $N(\mu, \sigma^2)$ with known σ^2 are

(i) $\bar{x} \pm 1 \cdot 96 \frac{\sigma}{\sqrt{n}}$

(ii) $\bar{x} \pm 2 \cdot 58 \frac{\sigma}{\sqrt{n}}$

(iii) $\bar{x} \pm 1 \cdot 28 \frac{\sigma}{\sqrt{n}}$

(iv) None of the above

(h) The necessary and sufficient condition for the existence of minimum variance unbiased estimator (MVUE) is that likelihood function can be expressed in the form

(i) $\frac{\partial}{\partial\theta}\log L = \frac{t-\theta}{\lambda}$

(ii) $\frac{\partial}{\partial\theta}\log L = \frac{t+\theta}{\lambda}$

(iii) $\frac{\partial}{\partial\theta}\log L = \frac{\theta-\lambda}{t}$

(iv) None of the above

2. Answer the following in brief : $2\times8=16$

(a) Distinguish between estimator and statistic with examples.

(b) State the sufficient condition for consistency.

(c) Define unbiased estimator and consistent estimator.

(d) State Cramer-Rao inequality and mention its uses.

(e) State the invariance property of maximum likelihood estimator (MLE).

(f) Distinguish between method of moment and maximum likelihood method as methods of estimation.

(g) Define confidence interval and confidence coefficient.

(h) Distinguish between point estimation and interval estimation.

3. (a) (i) Describe clearly the criterion of a good estimator. 5

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample of size n from a Poisson population with parameter λ. Find the unbiased estimators of λ and λ^2. Also find the variance of the estimator of λ. 5

Or

(b) (i) Explain the minimum variance unbiased estimator (MVUE) with examples. If t is an unbiased estimator of θ, find Cramer-Rao bound of it. 5

(ii) Establish a necessary and sufficient condition for an unbiased estimator to be MVU estimator. Give the statement of the factorization theorem and Rao-Blackwell theorem. 5

4. (a) (i) Define efficiency of an estimator. Let $x_1, x_2, \cdots, x_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Consider two estimators of μ as

$$T_1 = \frac{1}{n}\sum^{n} x \text{ and } T_2 = \frac{1}{n+1}\sum^{n} x$$

Find the relative efficiency of T_2 compared to T_1. 1+4=5

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample of observations from $N(\mu, \sigma^2)$. Show that the sample mean $\bar{x}$ and sample variance S^2 are consistent estimators of μ and σ^2 respectively. 5

Or

(b) (i) What is a sufficient statistic? Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from population having p.d.f.

$$f(x) = \theta x^{\theta - 1};\ 0 < x < 1,\ \theta > 0$$

Find a sufficient statistic for θ. 5

(ii) Prove that for Cauchy's distribution, not sample mean but sample median is a consistent estimator of the population mean. 5

5. State the important properties of maximum likelihood estimator. 5

6. (a) (i) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations with Poisson population with parameter θ. Find the maximum likelihood estimator of θ. 5

(ii) Explain the principle of maximum likelihood for estimation of population parameters. 5

Or

(b) (i) Write an explanatory note on the method of minimum chi-square in the theory of estimation. 5

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from a population having p.d.f.

$$f(x) = \theta e^{-\theta x}, \; 0 \le x \le \infty$$

Find the estimators of θ, θ^2 and θ^3 by the method of moment. 5

7. (a) Write short notes on the following : 4+4=8

(i) Method of least squares

(ii) Method of moment

Or

(b) Discuss the necessity of estimation of population parameters. What are the different methods of estimation? Which method would you recommend to have a good estimator? Write the reason why. 2+4+2=8

8. Write an explanatory note on interval estimation. 5

9. (a) Obtain $100(1-\alpha)\%$ confidence interval for the parameters μ and σ^2 of the normal distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2};$$

$$-\infty < x < \infty$$
$$-\infty < \mu < \infty$$
$$\sigma > 0$$ 8

Or

(b) Give the concepts of confidence intervals for large samples. Find $100(1-\alpha)\%$ confidence interval of p in $b(n, p)$ using large sample. 8

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) Unbiasedness is a property associated with

(i) finite sample size n

(ii) infinite sample size n

(iii) both finite and infinite sample size

(iv) None of the above

(b) Bias of an estimator can be

(i) positive

(ii) negative

(iii) either positive or negative

(iv) always zero

(c) If T_n and T_n' are two unbiased estimators of $\gamma(\theta)$ based on the random sample $X_1, X_2, \cdots, X_n$, then T_n is said to be UMVUE if and only if

(i) $V(T_n) \geq V(T_n')$ *(ii)* $V(T_n) \leq V(T_n')$

(iii) $V(T_n) = V(T_n')$ *(iv)* $V(T_n) = V(T_n') = 1$

(d) Minimum chi-squared estimators are not necessarily

(i) efficient

(ii) consistent

(iii) unbiased

(iv) All of the above

(e) Generally, the estimators obtained by the method of moments as compared to ML estimators are

(i) less efficient

(ii) more efficient

(iii) equally efficient

(iv) None of the above

(f) Let θ be an unknown parameter and T_1 be an unbiased estimator of θ. If $\text{var}(T_1) \le \text{var}(T_2)$ for T_2 to be any other unbiased estimator, then T_1 is known as

(i) minimum variance unbiased estimator

(ii) unbiased and efficient estimator

(iii) consistent and efficient estimator

(iv) unbiased, consistent and minimum variance estimator

(g) The method of maximum likelihood estimation was initially formulated by

(i) C. F. Gauss

(ii) R. A. Fisher

(iii) H. Cramer and C. R. Rao

(iv) C. R. Rao

(h) 95% confidence limit for θ in case of large sample n of the density function

$$f(x, \theta) = \theta e^{-\theta x},\ 0 < x < \infty$$

is

(i) $\left(1 \pm \frac{1 \cdot 96}{\sqrt{n}}\right)\bar{x}$

(ii) $\left(1 \pm \frac{1 \cdot 96\bar{x}}{\sqrt{n}}\right)\Big/\bar{x}$

(iii) $\left(1 \pm \frac{1 \cdot 96}{\sqrt{n}}\right)\Big/\bar{x}$

(iv) None of the above

2. Answer the following in brief : 2×8=16

(a) If T_n is an unbiased estimator of θ, then show that T_n^2 is a biased estimator of θ^2.

(b) Mention the criterion that should be satisfied by a good estimator.

(c) Distinguish between the method of moments and maximum likelihood as techniques of point estimation.

(d) Define efficiency of an estimator.

(e) State the sufficient condition for consistency.

(f) "MLE's are always consistent estimators but need not be unbiased." Justify the statement with an example.

(g) State the factorization theorem on sufficiency.

(h) What are meant by *(i)* 95% and *(ii)* 99% confidence limits and confidence intervals for the estimation of the population mean?

3. *(a)* *(i)* Define the following terms and give one example for each : 5

(1) Consistent statistic

(2) Unbiased statistic

(3) Sufficient statistic

(4) Efficiency

(5) Estimate

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample from $N(\mu, \sigma^2)$. Find the sufficient estimators for μ and σ^2. 5

Or

(b) *(i)* What is meant by estimation? State the different methods of estimation. 5

(ii) If t_1 and t_2 be two independent and unbiased estimators of θ, find the unbiased estimators of (1) θ^2 and (2) $\theta(\theta-1)$. 5

4. *(a)* *(i)* Define MVU estimator. Show that an MVU estimator is always unique. 2+3=5

(ii) If T_1 is an MVU for θ and T_2 is any other unbiased estimator of θ, then prove that no linear combination of T_1 and T_2 is an MVU estimator. 5

Or

(b) *(i)* Write an explanatory note on Rao-Blackwell theorem. 5

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample from a uniform population $[0, \theta]$. Find a sufficient estimator for θ. 5

5. State the important properties of maximum likelihood method of estimation. 5

6. (a) (i) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from Bernoulli population with parameter p. Find the ML estimators of (1) p and (2) p^2. 5

(ii) Show that ML estimator is consistent. 5

Or

(b) (i) Write an explanatory note on the method of minimum variance in the theory of estimation. 5

(ii) Explain the method of minimum chi-square in the theory of estimation. 5

7. (a) Describe some important characteristics of method of moment. Find the estimation of θ in Poisson distribution by the method of moment. 4+4=8

Or

(b) Write short notes on the following : 4×2=8

(i) Method of least squares

(ii) Cramer-Rao inequality

8. Distinguish between point estimation and interval estimation. Find the 95% confidence limit and confidence interval for population mean μ of normal distribution. 2+3=5

9. *(a)* Obtain $100(1-\alpha)$ percent confidence interval for the parameters μ and σ^2 of the normal distribution

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < \mu < \infty; \sigma > 0$$
$$-\infty < x < \infty$$ 8

Or

(b) Write an explanatory note on interval estimation. A random sample of size 100 has mean 15 and the population variance 25. Find the interval estimate of population mean with confidence levels of 95% and 99%. 4+4=8

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