Total No. of Printed Pages—12

5 SEM TDC STS M 2 (N/O)

2018

(November)

STATISTICS

(Major)

Course : 502

(Testing of Hypothesis)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

1. Fill in the blanks : 1×8=8

(a) A wrong decision about the null hypothesis leads to _____ types of error.

(b) If β is the probability of type–II error, then $(1-\beta)$ is called _____ of the test.

(c) Every most powerful (MP) or uniformly most powerful (UMP) critical region (CR) is necessarily _____.

(d) From the normal probability table we know that P (_____ $\leq Z \leq$ _____) = 0·9973.

(e) The significance of an observed multiple correlation coefficient can be tested by _____ test.

(f) The number of elements in a run is usually called the _____ of the run.

(g) _____ methods are referred to as distribution-free methods.

(h) Mann-Whitney's U-test is the best non-parametric test for _____.

2. Answer the following questions : 2×8=16

(a) Explain the best critical region.

(b) Identify the composite hypotheses in the following, where μ is the mean and σ^2 is the variance of the distribution :

(i) $H_0: \mu \leq 0, \sigma^2 = 1$

(ii) $H_0: \mu = 0, \sigma^2 = 0$

(Continued)

(iii) $H_0: \mu \leq 0, \sigma^2$ = arbitrary

(iv) $H_0: \sigma^2 = \sigma_0^2$ (a given value), μ arbitrary.

(c) What is power function of a test?

(d) Describe the steps which are followed in the normal test for testing the significance for large samples.

(e) What is the paired *t*-test for testing the significance of difference of means?

(f) What do you understand by non-parametric methods of testing the hypothesis?

(g) State the assumptions made in sign test.

(h) What are the drawbacks of non-parametric tests?

3. (a) Define two types of error, level of significance and power of a test, with reference to testing of a hypothesis. State clearly the theorem which is used to determine the best critical region for simple hypothesis at a given significance level. 4+2+2+4=12

Or

(b) Let $x_1, x_2, \cdots, x_n$ be a random sample from a normal population with mean μ and variance σ^2, μ and σ^2 being unknown. We wish to test $H_0: \mu = \mu_0$ (specified) against $H_1: \mu \neq \mu_0$, $0 < \sigma^2 < \infty$.

Show that the likelihood-ratio test is the same as the two-tailed *t*-test. 12

4. *(a)* What is meant by a statistical hypothesis? Show that a most powerful test is necessarily unbiased. 2+7=9

Or

(b) Show that the likelihood-ratio test for testing the equality of variances of two normal distributions is the usual *F*-test. 9

5. *(a)* Explain how you would proceed to test the difference between the sample and population means when the sample is large. Given that $\overline{X} = 80$, $\sigma = 15$, $n = 60$, test the hypothesis that $\mu = 85$ against $\mu \neq 85$. 5+5=10

Or

(b) What are the assumptions made in the Student's *t*-test? Mention some applications of Student's-*t* as the test of significance. Explain the *t*-test for single mean. Find the Student's-*t* for the following variate values in a sample of eight : $-4, -2, -2, 0, 2, 2, 3, 3$, taking the mean of the universe to be zero. How would you proceed further? 2+2+2+4=10

6. *(a)* Describe the χ^2-test for independence of attributes, stating clearly the conditions for its validity. Give a rule for calculating the number of degrees of freedom to be assigned to χ^2. Illustrate your answer with an $m \times n$ contingency table, explaining the null hypothesis that is being tested. 8

Or

(b) Discuss the application of *F*-test in testing if the two variances are homogeneous. Explain why the larger variance is placed in the numerator of the *F*-statistic. 5+3=8

7. *(a)* Describe the procedure in median test when there are two independent samples. What non-parametric test would you use when the two samples are related? 8+2=10

Or

(b) Develop the Mann-Whitney-Wilcoxon test and obtain the mean and variance of the test statistic T. How is the test carried out for large samples? 8+2=10

8. *(a)* Derive the sign test. 7

Or

(b) Ten coins are tossed and the number of heads (H) and tails (T) are found as follows :

$$H\ T\ H\ T\ T\ T\ H\ H\ H\ H$$

Do you think that the coin-tossing experiment is done randomly? 7

(Continued)

(Old Course)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

1. Choose the correct alternatives from the following : 1×8=8

(a) When a null hypothesis is accepted, it is possible that

(i) a correct decision has been made

(ii) a type I error has been made

(iii) Both *(i)* and *(ii)* have occurred

(iv) Neither *(i)* nor *(ii)* has occurred

(b) Neyman-Pearson lemma provides

(i) a most powerful test

(ii) an unbiased test

(iii) an admissible test

(iv) a minimax test

of simple hypothesis against a simple alternative hypothesis.

(Turn Over)

(c) A test T, which is at least as powerful as any other test of the same size is called

(i) best test

(ii) most powerful test

(iii) uniformly most powerful test

(iv) None of the above

(d) Student's t-test was introduced by

(i) J. Medhi

(ii) S. Biswas

(iii) W. S. Gosset

(iv) W. S. Cochran

(e) The hypothesis is that the population variance has a specified value can be tested by

(i) F-test

(ii) Z-test

(iii) χ^2-test

(iv) None of the above

(f) The size of the critical region is

(i) α

(ii) β

(iii) $1-\beta$

(iv) $1-\alpha$

(g) SPRT provides for a minimum amount of sampling and thus result is considerable saving in terms of

(i) inspection

(ii) time

(iii) money

(iv) All of the above

(h) Most of the non-parametric methods utilize measurements on

(i) interval scale

(ii) ratio scale

(iii) ordinal scale

(iv) nominal scale

2. Answer the following in brief : 2×8=16

(a) Define simple hypothesis and composite hypothesis.

(b) What are type–I and type–II errors?

(c) Define level of significance and power of a test.

(d) Define most powerful test.

(e) When do you use paired t-test?

(f) Define likelihood-ratio test.

(g) State how the sequential test differs from Neyman-Pearson test.

(h) State clearly the hypothesis which is tested in run test for one sample problem.

3. (a) State and prove Neyman-Pearson fundamental lemma for testing a simple null hypothesis against a simple alternative hypothesis. 8

(b) Given the frequency function

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} , & 0 \le x \le \theta \\ 0 , & \text{otherwise} \end{cases}$$

and you are testing the null hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$ by means of a single observed value of x. What would be the probabilities of the Type–I and Type–II errors, if you choose the interval (i) $0 \cdot 5 \le x$ and (ii) $1 \le x \le 1 \cdot 5$ as the critical region? Also obtain the power function of the test. 8

Or

(c) Show that the power of a best critical region for testing a simple null hypothesis against a simple alternative hypothesis is never less than its size. 8

4. Describe any *two* of the following tests of significance : 4×2=8

(a) Chi-square test of goodness of fit

(b) Student's *t*-test

(c) Large sample test for the difference of two sample proportions

5. Discuss the general method of construction of likelihood-ratio test. Under what circumstances would you recommend this test? 4+2=6

6. (a) Give in brief the idea of SPRT. 6

(b) Develop SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$, where θ_0 is the parameter of a Poisson distribution. Write down the approximate expression for OC and ASN functions of the test. 8

Or

(c) Define OC and ASN functions in sequential analysis. Write down their approximate expressions for the SPRT of a simple hypothesis against a simple alternative hypothesis. 8

7. (a) Distinguish between parametric test and non-parametric test. What is the advantage of non-parametric test? 5

(b) Describe the median test, when there are K independent samples. 7

Or

(c) Describe briefly Kolmogorov-Smirnov test for goodness of fit. 7

★★★