3 SEM TDC STS M 1 (N/O)

2018

(November)

STATISTICS

(Major)

Course : 301

(Probability and Distribution—I)

(New Course)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the given alternatives : 1×8=8

(a) Classical probability is possible in case of

(i) unequilikely outcomes

(ii) equilikely outcomes

(iii) either equilikely or unequilikely outcomes

(iv) None of the above

(b) A, B and C are three arbitrary events. The expression for the events 'at least two occur' is

(i) $A \cap B \cap \overline{C}$

(ii) $A \cap \overline{B} \cap C$

(iii) $(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup$
$(\overline{A} \cap B \cap C) \cup (A \cap B \cap C)$

(iv) None of the above

(c) A coin is tossed three times in succession, then the number of sample points in the sample space is

(i) 6

(ii) 8

(iii) 3

(iv) None of the above

(d) Two random variables X and Y are uncorrelated if

(i) $E(XY) = E(X) \cdot E(Y)$

(ii) $V(X) = V(Y)$

(iii) $-1 \leq r_{XY} \leq 1$

(iv) None of the above

(e) Regression lines of Y on X and X on Y are perpendicular to each other if correlation between X and Y is

(i) 0

(ii) 1

(iii) −1

(iv) None of the above

(f) If X_1 and X_2 are independent random variables, then $M_{X_1+X_2}(t)$ is equal to

(i) $M_{X_1}(t)+M_{X_2}(t)$

(ii) $M_{X_1}(t)-M_{X_2}(t)$

(iii) $M_{X_1}(t)\cdot M_{X_2}(t)$

(iv) None of the above

(g) Both the regression coefficients

(i) must be less than 1

(ii) may be less than 1

(iii) should be of the same sign

(iv) may be greater than 1

(h) If X is a random variable, then $E(t^x)$ is known as

(i) characteristic function

(ii) moment-generating function

(iii) probability-generating function

(iv) the xth moment

2. Answer the following in brief : 2×8=16

(a) Write down the statistical definition of probability.

(b) Three coins are tossed. Express the following events by appropriate sets :

(i) A : The event of getting at least two heads

(ii) B : The event of getting at most two heads

Examine whether A and B are mutually exclusive.

(c) Find the moment-generating function of the random variable whose moments are given by $\mu_r' = \underline{|(r+1)} \cdot 2^r$.

(d) Define characteristic function and give its importance.

(e) A number is chosen from each of the two sets

(1, 2, 3, 4, 5, 6, 7, 8, 9); (4, 5, 6, 7, 8, 9)

If P_1 is the probability that the sum of the two numbers be 10 and P_2 the probability, their sum be 8, find $P_1 + P_2$.

(f) What are meant by marginal distribution and conditional distribution?

(g) What is meant by regression? If X and Y are two random variables, write down the two regression lines.

(h) If X is a random variable and a and b are constants, then show that

$$V(aX + b) = a^2 V(X)$$

3. *(a)* If $B < A$, then prove that—

(i) $P(A \cap \overline{B}) = P(A) - P(B)$;

(ii) $P(B) \leq P(A)$. 4

Or

(b) A card is drawn from a well-shuffled pack of cards. What is the probability that it is either a spade or an ace? State the theorem that applies here. 4

4. *(a)* In 2020, there will be three candidates for the position of principal of a college in Dibrugarh—Mr. *X*, Mr. *Y* and Mr. *Z*, whose chances of getting the appointment are in the proportion 4 : 2 : 3 respectively. The chance that Mr. *X* if selected will introduce coeducation in the college is 0·3. The chances of Mr. *Y* and Mr. *Z* doing the same are 0·5 and 0·8. What is the chance that there will be coeducation in the college in 2020? State the theorem that applies here. 5+2=7

Or

(b) (i) An urn contains 10 white and 10 black balls. Another urn contains 5 white and 5 black balls. One ball is transferred from the first urn to the second urn and then a ball is drawn from the later. What is the probability that it is white? 4

(ii) The probability of solving a problem by A and B are respectively $\frac{1}{10}$ and $\frac{1}{12}$. What is the probability that the problem will be solved if it is assigned to them? 3

5. (a) Distinguish between correlation analysis and regression analysis. 3

(b) The joint probability distribution of a pair (X, Y) of random variables is given by the table below :

$X \rightarrow$ $Y \downarrow$	1	2	3
1	0·1	0·2	0·2
2	0·2	0·3	0·1

Find—

(i) the marginal distribution;

(ii) the conditional distribution of X, given $Y = 1$;

(iii) $P\{(X + Y) < 4\}$. 5

(c) Two random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & , \text{otherwise} \end{cases}$$

Find—

(i) the marginal probability density function of X and Y;

(*ii*) the conditional density functions;

(*iii*) $\text{var}(X)$ and $\text{var}(Y)$;

(*iv*) the covariance between X and Y. 6

6. (*a*) Define distribution function. A random variable X has a continuous distribution function $F(x)$, where

$$F(x) = \begin{cases} 0 & \text{if} \quad x \le 0 \\ kx & \text{if} \quad 0 \le x \le 1 \\ 1 & \text{if} \quad x > 1 \end{cases}$$

(*i*) Determine constant k.

(*ii*) Compute the probabilities $P\left[X = \frac{1}{2}\right]$ and $P\left[X < \frac{1}{2}\right]$. 3

(*b*) Define a random variable and its mathematical expectation. Show that the mathematical expectation of sum of two random variables is the sum of their individual expectations. Can it be extended? 1+2+3+1=7

7. (*a*) Define moment-generating function of a random variable. A random variable X has the probability function

$$P(X = x) = \frac{1}{2^x}; \quad x = 1, 2, 3, \ldots$$

Find its moment-generating function and hence mean and variance. 6

Or

(*b*) Define cumulant-generating function. Show that the rth cumulant for the distribution

$$f(x) = ke^{-kx};\ 0 < x < \infty,\ k \neq 0 \in R$$

is $\frac{1}{k^r}\lfloor(r-1)$. 6

8. (*a*) State some important properties of characteristic function. 4

(*b*) If X is some random variable with characteristic function $\phi_X(t)$ and if $\mu'_r = E(X^r)$ exists, then prove that

$$\mu'_r = (-1)^r \left|\frac{\partial^r}{\partial t^r}\phi(t)\right|_{t=0}$$ 5

9. (*a*) Define probability-generating function of a random variable. Establish its relationship with the moment-generating function. If $P(s)$ is the probability-generating function of the random variable X, then find the generating function for $\frac{X-a}{b}$; a, b are constants. 6

Or

(*b*) Obtain the probability-generating function of a random variable X with distribution $P(X) = \frac{e^{-\lambda}\lambda^x}{\lfloor x}$; $x = 0, 1, \ldots, \infty$ and with the help of it, find the mean of the variable X. 6

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the given alternatives : $1\times8=8$

(a) Classical probability is possible in case of

(i) unequilikely outcomes

(ii) equilikely outcomes

(iii) either equilikely or unequilikely outcomes

(iv) None of the above

(b) A, B and C are three arbitrary events. The expression for the events 'at least two occur' is

(i) $A \cap B \cap \overline{C}$

(ii) $A \cap \overline{B} \cap C$

(iii) $(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C) \cup (A \cap B \cap C)$

(iv) None of the above

(c) A coin is tossed three times in succession, then the number of sample points in the sample space is

(i) 6

(ii) 8

(iii) 3

(iv) None of the above

(d) Two random variables X and Y are uncorrelated if

(i) $E(XY) = E(X) \cdot E(Y)$

(ii) $V(X) = V(Y)$

(iii) $-1 \le r_{XY} \le 1$

(iv) None of the above

(e) Regression lines of Y on X and X on Y are perpendicular to each other if correlation between X and Y is

(i) 0

(ii) 1

(iii) −1

(iv) None of the above

(f) If X_1 and X_2 are independent random variables, then $M_{X_1+X_2}(t)$ is equal to

(i) $M_{X_1}(t) + M_{X_2}(t)$

(ii) $M_{X_1}(t) - M_{X_2}(t)$

(iii) $M_{X_1}(t) \cdot M_{X_2}(t)$

(iv) None of the above

(g) If b_{YX} and b_{XY} are the regression coefficients, then both of them

(i) must be less than 1

(ii) may be less than 1

(iii) should be of the same sign

(iv) may be greater than 1

(h) If X is a random variable, then $E(t^x)$ is known as

(i) characteristic function

(ii) moment-generating function

(iii) probability-generating function

(iv) the xth moment

2. Answer the following in brief : $2 \times 8 = 16$

(a) Write down the statistical definition of probability.

(b) Three coins are tossed. Express the following events by appropriate sets :

(i) A : The event of getting at least two heads

(ii) B : The event of getting at most two heads

Also examine whether A and B are mutually exclusive.

(c) Find the moment-generating function of the random variable whose moments are given by $\mu'_r = \underline{|(r+1)} \cdot 2^r$.

(d) Define characteristic function and give one of its applications.

(e) A number is chosen from each of the two sets

(1, 2, 3, 4, 5, 6, 7, 8, 9); (4, 5, 6, 7, 8, 9)

If P_1 is the probability that the sum of the two numbers be 10 and P_2 the probability, their sum be 8, find $P_1 + P_2$.

(f) Distinguish between correlation analysis and regression analysis.

(g) What is meant by conditional density function?

(h) If X is a random variable and a and b are constants, then show that

$$V(aX + b) = a^2 V(X)$$

3. (a) If $B < A$, then prove that—

(i) $P(A \cap \overline{B}) = P(A) - P(B)$;

(ii) $P(B) \leq P(A)$. 4

Or

(*b*) A card is drawn from a well-shuffled pack of cards. What is the probability that it is either a spade or an ace? State the theorem that applies here. 4

4. (*a*) (*i*) In a family, there are four children. Find the probability that there is exactly two girls, at most two girls and at least two girls, assuming the birth of boy and girl are equally likely. 3

(*ii*) An urn contains 9 balls, two of which are red, three are blue and four are black. Three balls are drawn from the urn at random. What is the probability that—

(1) three balls are of different colours;

(2) two balls are of the same colour and third of different colour;

(3) the balls are of the same colour? 4

Or

(*b*) In 2019, there will be three candidates for the position of principal of a college in Dibrugarh—Mr. X, Mr. Y and Mr. Z whose chances of getting the appointment are in the proportion 4 : 2 : 3 respectively. The chance that Mr. X if selected will introduce coeducation in the college is 0·3.

(*Turn Over*)

The chances of Mr. Y and Mr. Z doing the same are 0·5 and 0·8. What is the chance that there will be coeducation in the college in 2019? 7

5. (a) Define the covariance between two variables in terms of X and Y. What is the covariance between two variables when X and Y are independent? 3

(b) The joint probability distribution of a pair (X, Y) of random variables is given by the table below :

$X \rightarrow$ $Y \downarrow$	1	2	3
1	0·1	0·2	0·2
2	0·2	0·3	0·1

Find—

(i) the marginal distribution of X and Y;

(ii) the conditional distribution of X, given $Y = 1$;

(iii) $P\{(X+Y) < 4\}$. 5

(c) Two random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & ; \text{ otherwise} \end{cases}$$

Find—

(i) the marginal probability density functions of X and Y;

(ii) the conditional density function;

(iii) $\text{var}(X)$ and $\text{var}(Y)$;

(iv) the covariance between X and Y. 6

6. *(a)* Define distribution function. A random variable X has a continuous distribution function $F(x)$, where

$$F(x) = \begin{cases} 0 & \text{if} \quad x \le 0 \\ kx & \text{if} \quad 0 < x \le 1 \\ 1 & \text{if} \quad x > 1 \end{cases}$$

(i) Determine the constant k.

(ii) Compute the probabilities $P\left[X = \frac{1}{2}\right]$ and $P\left[X < \frac{1}{2}\right]$.

3

(b) Define a random variable and its mathematical expectation. Show that the mathematical expectation of sum of two random variables is the sum of their individual expectations. Can it be extended?

1+2+3+1=7

7. *(a)* Define moment-generating function of a random variable. A random variable X has the probability function

$$P(X = x) = \frac{1}{2^x}; \; x = 1, 2, 3, \ldots$$

Find its moment-generating function and hence mean and variance. 6

Or

(b) Define cumulant-generating function. Show that the rth cumulant for the distribution $f(x) = ke^{-kx}$; $0 < x < \infty$, $k \neq 0 \in R$ is $\frac{1}{k^r} \lfloor (r-1)$. 6

8. (a) State four important properties of characteristic function. 4

(b) If X is some random variable with characteristic function $\phi_X(t)$ and if $\mu_r' = E(X^r)$ exists, then prove that

$$\mu_r' = (-1)^r \left| \frac{\partial^r}{\partial t^r} \phi(t) \right|_{t=0}$$

5

9. (a) Define probability-generating function of a random variable. Establish its relationship with the moment-generating function. If $P(s)$ is the probability-generating function of the random variable X, then find the generating function for $\frac{X-a}{b}$; a, b are constants. 6

Or

(b) Obtain the probability-generating function of a random variable X with distribution $P(X) = \frac{e^{-\lambda}\lambda^x}{\lfloor x}$; $x = 0, 1, ..., \infty$ and with the help of it, find the mean of the variable X. 6

★★★