Total No. of Printed Pages—15

3 SEM TDC STS M 2 (N/O)

2018

(November)

STATISTICS

(Major)

Course : 302

(Numerical Methods)

(New Course)

Full Marks : 80
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) The value of $\Delta^4(1-x)(1-2x)(1-3x)$ is

(i) –6

(ii) 0

(iii) –36

(iv) 24

(b) Which one of the following is not true?

(i) $\Delta y_2 = y_3 - y_2$

(ii) $\nabla y_2 = y_2 - y_1$

(iii) $\nabla^2 y_2 = y_2 - 2y_1 + y_0$

(iv) $\Delta^2 y_2 = y_3 - 2y_2 + y_1$

(c) If Δ, E, δ are forward difference operator, shift operator and central difference operator respectively, then which one of the following relations is not true?

(i) $\Delta = \delta E^{\frac{1}{2}}$

(ii) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

(iii) $\delta = \Delta E^{-\frac{1}{2}}$

(iv) $\delta = E^{-\frac{1}{2}} - E^{\frac{1}{2}}$

(d) Stirling's central difference formula is the mean of

(i) backward and forward formulae of Newton

(ii) backward and forward formulae of Gauss

(iii) forward formulae of Gauss and Newton

(iv) forward formulae of Newton

(e) The nth divided differences of a polynomial of nth degree are

(i) always zero

(ii) always equal to n

(iii) always constant

(iv) not defined

(f) Simpson's $\frac{1}{3}$rd rule is derived under the assumption that the integrand is a polynomial of degree

(i) one

(ii) two

(iii) three

(iv) n

(g) $f(x) = 2 - 3x + 4x^4 + 2x^5$ is a polynomial of

(i) degree 5 with 4 roots

(ii) degree 3 with 3 roots

(iii) degree 5 with 3 roots

(iv) degree 5 with 5 roots

(h) Which one of the following is a transcendental function?

(i) $x^2 - 7\log 2$

(ii) $5x + e^x(x+2)$

(iii) $3x^2 + x + \cos^2 6$

(iv) $4x^2 + 9x + 7\log 4 + 2e^{2\cdot 5}$

2. Answer any *two* of the following : 4×2=8

(a) Prove that

$$e^x = \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$$

The interval of differencing being 1.

(b) Prove that

$$\Delta^n (ab^{cx}) = \Delta^{n-1} \cdot \Delta (ab^{cx})$$

The interval of differencing being unity.

(c) Given

$f(0) = 3$, $f(1) = 12$, $f(2) = 81$,
$f(3) = 200$, $f(4) = 100$, $f(5) = 8$

Find $\Delta^5 f(0)$.

(Continued)

3. *(a)* Express $2x^3 - 3x^2 + 3x - 10$ and its successive differences in factorial notation. 5

Or

(b) Use the method of finite differences to find the sum of the series

$$\sum_{x=1}^{n} U_x$$

where

$$U_x = \frac{x+3}{x(x+1)(x+2)}$$ 5

4. *(a)* If

$$f(x) = \frac{1}{x^2}$$

find divided differences $f(a, b)$ and $f(a, a)$. 4

(b) Given

$$f(0) = 3,\ f(1) = 6,\ f(2) = 11,$$
$$f(3) = 18,\ f(4) = 27$$

Determine the form of the function $f(x)$. 4

5. Answer any *three* of the following : 8×3=24

(a) What are meant by interpolation and extrapolation? Write a note on four different applications of interpolation techniques. 3+5=8

(b) Derive Netwton's formula for forward interpolation and mention two of the limitations of the technique. 6+2=8

(c) In what situation(s) the formula for central differences are recommended? Write down the Gauss backward formula for central differences and hence interpolate the population of the year 1996 from the following data, where population (in thousand) are given for different years : 1+7=8

Year	1961	1971	1981	1991	2001	2011
Population	12	15	20	27	39	52

(d) What are divided differences? Prove that the divided differences are symmetric functions of their arguments. 2+6=8

(Continued)

(e) What is inverse interpolation? Explain. Write down the Lagrange's formula for inverse interpolation and hence or otherwise find the value of x, when $y = f(x) = 7$. Given

x	1	3	4
$y = f(x)$	4	12	19

2+6=8

6. Answer any *three* of the following : 9×3=27

(a) When do we use 'numerical differentiation'? How do the errors occur in numerical computation of derivatives? Given

x	0	1	3	4
$f(x)$	1	4	40	85

Find $f'(x)$ and $f''(x)$ at $x = 2$. 1+2+6=9

(b) Derive trapezoidal formula from general quadrature. Given the values of x and $f(x) = e^{\sin x}$ as

x	0	$\pi/6$	$\pi/3$	$\pi/2$
$f(x)$	1	1·6487	2·4596	2·7183

Find an approximate value of

$$\int_0^{\pi/2} f(x)\,dx$$

using Simpson's rule. 5+4=9

(c) Describe regula falsi method and solve $x^3 - 2x - 5 = 0$ for a root lying between 2 and 3 (do up to third approximation only). 5+4=9

(d) Write short notes on—

(i) Weddle's rule for numerical integration;

(ii) Newton-Raphson method and its limitations. 4½+4½=9

(Old Course)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) The *n*th forward difference is given by

(i) $\Delta^n f(x+h) - \Delta^{n-1} f(x)$

(ii) $\Delta^{n+1} f(x+h) - \Delta^n f(x)$

(iii) $\Delta^{n-1} f(x+h) - \Delta^n f(x)$

(iv) $\Delta^{n-1} f(x) - \Delta^{n-1} f(x+h)$

(b) Which one of the following is not true?

(i) $E \equiv 1 + \Delta$

(ii) $\nabla = 1 - E^{-1}$

(iii) $\nabla E = \Delta$

(iv) $\nabla = 1 + E^{-1}$

(c) Divided differences are useful, when

(i) arguments are equally spaced

(ii) arguments are unequally spaced

(iii) arguments are advanced with unit intervals

(iv) None of the above

(d) Iterative method of inverse interpolation utilizes

(i) Newton's Forward Formula only

(ii) Newton's Backward Formula only

(iii) Any suitable polynomial formula

(iv) None of the above

(e) If n entries are known for $(n+1)$ arguments at equal interval of differencing having a missing entry, the non-polynomial interpolation method is

(i) Lagrange's method

(ii) Newton's method

(iii) Binomial expansion method

(iv) All of the above

(f) Which of the following is true?

(i) $\mu^2 = 1 + \delta^2$

(ii) $E^{1/2} = \mu - \frac{1}{2}\delta$

(iii) $\delta = \Delta E^{1/2}$

(iv) $E^{-1/2} = \mu - \frac{1}{2}\delta$

(Continued)

(g) Simpson's one-third rule is called

(i) straight line formula

(ii) hyperbolic formula

(iii) parabolic formula

(iv) None of the above

(h) Which of the following is not a transcendental equation?

(i) $2x - \log_{10} 2 = 7$

(ii) $2x - \log_{10} x = 7$

(iii) $e^x - 3x = 0$

(iv) $e^{-x} = \sin x$

2. Answer the following in brief : 2×8=16

(a) Establish the relation

$$E^n = (1 + \Delta)^n$$

(b) Evaluate

$$\Delta\left[\frac{2x}{2x+1}\right]$$

The interval of differencing being unity.

(c) What is meant by interpolation? Write down the underlying assumptions.

(d) State Gauss's forward interpolation formula and mention its characteristics.

(e) What is the difference between Lagrange's method and Newton's method as a method of interpolation?

(f) What are the basic conditions to apply Simpson's one-third rule?

(g) What are algebraic and transcendental equations? Give example.

(h) In which situation, the bisection method is applicable?

3. (a) (i) Discuss, if operators E and Δ obey the distributive, commutative and associative laws of algebra. 6

(ii) Evaluate $\Delta^2 \log x$. 3

Or

(b) (i) Use the method of finite differences to sum the series

$$1^3 + 2^3 + 3^3 + \cdots + n^3$$ 6

(ii) Sum to n terms the series whose xth term is $x(x-1)(x-2)$. 3

(Continued)

4. Answer any *two* of the following : 13×2=26

(a) *(i)* Derive the Newton-Gregory forward interpolation formula for equal intervals. 5

(ii) Given the following data :

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

Determine the form of the function $f(x)$. 4

(iii) What are backward differences of a polynomial? State the relationship of backward operator ∇ and E operator. 4

(b) What are divided differences? Prove that the value of any divided difference is independent of the order of the argument. Establish the relation between divided differences and ordinary differences. If

$$f(x) = \frac{1}{x^2}$$

find the divided differences of $f(a, b)$.

2+5+4+2=13

(c) (i) Derive the Gauss backward interpolation formula for equal intervals. 5

(ii) State Stirling's formula. 2

(iii) Apply a central difference formula to obtain $f(25)$, given that $f(20) = 14$, $f(24) = 32$, $f(28) = 35$, $f(32) = 40$. 6

(d) (i) Deduce Lagrange's formula for interpolation. 6

(ii) Find the form of the function, given that

x	0	1	2	5
$f(x)$	2	3	12	147

5

(iii) In which situation, Lagrange's interpolation formula is used in general? 2

5. Answer any *three* of the following : 7×3=21

(a) What is numerical differentiation? Find the first two derivatives of $f(x)$ at $x = 1$ from the following table : 2+5=7

x	−2	−1	0	1	2	3	4
$f(x)$	104	17	0	−1	8	69	272

(Continued)

(b) Using Newton's polynomial, derive a general quadrature formula for equi-distant ordinates. Hence obtain the Simpson's $\frac{3}{8}$th rule. 4+3=7

(c) Describe Newton-Raphson method. In which situation this method is applicable? 5+2=7

(d) Describe bisection method and mention clearly how the percentage error is used to determine the number of iterations. 4+3=7

(e) Write short notes on the following : 3½+3½=7

(i) Weddle's rule

(ii) Regula falsi method

(f) Discuss briefly the method of iteration for solving transcendental equation. Find by iteration method, a real root of $2x - \log_{10} x = 7$ correct to four places of decimals. 4+3=7

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