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5 SEM TDC STS M 1 (N/O)

2017

(November)

STATISTICS

(Major)

Course : 501

(Estimation)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) T is an asymptotically unbiased estimator of θ, if

(i) $\lim_{n \to \infty} E(T) = \theta$

(ii) $E(T) = \theta$

(iii) $E(T) > \theta$

(iv) $\lim_{n \to \infty} E(T) \neq \theta$

(*b*) Consistency of an estimator T_n of $\gamma(\theta)$ means

(*i*) $P_\theta\{|T_n - \gamma(\theta)| > \varepsilon\} = 1$

(*ii*) $\lim_{n \to \infty} P_\theta\{|T_n - \gamma(\theta)| > \varepsilon\} = 1$

(*iii*) $\lim_{n \to \infty} P_\theta\{|T_n - \gamma(\theta)| < \varepsilon\} = 0$

(*iv*) $\lim_{n \to \infty} P_\theta\{|T_n - \gamma(\theta)| > \varepsilon\} = 0$

(*c*) Bias of an estimator can be

(*i*) positive

(*ii*) negative

(*iii*) either positive or negative

(*iv*) always zero

(*d*) If $\hat{\theta}_1$ is the most efficient estimator with variance V_1 and $\hat{\theta}_2$ is other estimator with variance V_2, then the efficiency E of $\hat{\theta}$ is defined as

(*i*) $E = \frac{V_2}{V_1}$

(*ii*) $E = \frac{V_1}{V_2}$

(*iii*) $E = V_1 - V_2$

(*iv*) $E = V_1 + V_2$

(e) A maximum likelihood estimator is always

(i) unbiased

(ii) consistent

(iii) unbiased and consistent

(iv) consistent but need not be unbiased

(f) Generally, the estimators obtained by the method of moments as compared to ML estimators are

(i) less efficient

(ii) more efficient

(iii) equally efficient

(iv) None of the above

(g) The formula for obtaining 95% confidence limit for the mean μ of a normal distribution $N(\mu, \sigma^2)$ with known σ is

(i) $-1 \cdot 96 \le \frac{\bar{x}-\mu}{\delta / \sqrt{n}} \le 1 \cdot 96$

(ii) $\bar{x} \pm 1 \cdot 96 \frac{\sigma}{\sqrt{n}}$

(iii) $P\left(-z_{\alpha/2} \le \frac{\bar{x}-\mu}{\sigma / \sqrt{n}} \le z_{\alpha/2}\right) = 0 \cdot 95$

(iv) All the above

(h) The method of maximum likelihood estimation was initially formulated by

(i) C. F. Gauss

(ii) R. A. Fisher

(iii) Cramer and Rao

(iv) C. R. Rao

2. Answer the following in brief : $2\times8=16$

(a) Mention the important properties to be possessed by a good estimator.

(b) Prove that if $T(X_1, X_2, \ldots, X_n)$ be an unbiased estimator of θ, it does not necessarily mean that T^2 will be an unbiased estimator for θ^2.

(c) Distinguish between the method of moments and maximum likelihood as technique of point estimation.

(d) State Cramer-Rao inequality and mention Cramer-Rao lower bound for the variance of the unbiased estimator.

(e) Write the statement of the factorization theorem. What is the most general form of the distribution admitting sufficient statistic?

(f) Write the difference between point estimation and interval estimation.

(g) State two important properties of maximum likelihood estimators.

(h) What do you mean by *(i)* 95% and *(ii)* 99% confidence limits and confidence intervals for the estimation of the population mean?

3. *(a)* *(i)* Define the following terms and give one example for each : 5

(1) Consistent statistic

(2) Unbiased statistic

(3) Sufficient statistic

(4) Efficiency

(5) Estimate

(ii) Prove that if T is a consistent estimator of θ so also T^2 is a consistent estimator of θ^2. 5

Or

(b) *(i)* What is the necessary and sufficient condition for T to be sufficient estimator for θ? Let $x_1, x_2, \ldots, x_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for θ. 5

(ii) What is most efficient estimator? If T_1 and T_2 are unbiased estimator of θ and $\text{var}(T_1) = 0{\cdot}2$ and $\text{var}(T_2) = 0{\cdot}6$, compute the relative efficiency of T_1 as compared to T_2. 5

4. *(a)* *(i)* Explain with illustration the concept of a minimum variance unbiased estimator (MVUE). 5

(ii) If T_1 and T_2 are two unbiased estimators of a parameter θ with variances σ_1^2 and σ_2^2 and correlation coefficient φ, then obtain the best linear combination of T_1 and T_2. 5

Or

(b) *(i)* If $X_1, X_2, \ldots, X_n$ be a random sample from a population with p.d.f. $f(x, \theta) = \theta \cdot x^{\theta-1}$, $0 < x < 1$, $\theta > 0$, then show that $t_i = \prod_{i=1}^{n} X_i$ is sufficient for θ. 5

(ii) Write a short note on Rao-Blackwell theorem. 5

5. State and explain the principle of maximum likelihood estimation. 5

6. *(a)* *(i)* Find the maximum likelihood estimator for P in the binomial probability distribution

$$f(x) = P^x (1-P)^{1-x}, \; x = 0, 1$$ 5

(ii) Prove that if a sufficient estimator exists, it is a function of maximum likelihood estimation. 5

Or

(b) *(i)* Describe the method of moments for estimating parameters. 5

(ii) Write an explanatory note on the method of least square in the theory of estimation. 5

7. *(a)* Write short notes on the following : 4×2=8

(i) Method of minimum variance

(ii) Method of minimum chi-square

Or

(b) Let $(X_1, X_2, ..., X_n)$ be a random sample from the p.d.f.

$$f(x, \theta) = \theta e^{-\theta x}, \quad 0 < x < \infty, \; \theta > 0$$
$$= 0, \quad \text{elsewhere}$$

Estimate θ using the method of moment. Describe some characters of the method of moment. 6+2=8

8. Give the concepts of confidence interval and confidence coefficient. 5

9. *(a)* Explain what is meant by interval estimation. In a random sample, out of 60 students, 40 students use spectacles. Find the 95% confidence limits for students using spectacles in the student population. 3+5=8

Or

(b) For the distribution $f(x, \theta) = \theta e^{-\theta x}$, $0 < x < \infty$. Obtain the $100(1-\alpha)\%$ confidence interval for θ (for large sample). 8

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) An estimator T_n is said to be sufficient statistic for a parameter $\gamma(\theta)$, if it contains all the information which is contained in the

(i) population

(ii) parameter $\gamma(\theta)$

(iii) sample

(iv) None of the above

(b) If T_n and T_n' are two unbiased estimators of $\gamma(\theta)$ based on the random sample $X_1, X_2, ..., X_n$, then T_n is said to be UMVUE if and only if

(i) $V(T_n) \geq V(T_n')$

(ii) $V(T_n) \leq V(T_n')$

(iii) $V(T_n) = V(T_n')$

(iv) $V(T_n) = V(T_n') = 1$

(c) Method of moment was first introduced by

(i) Karl Pearson

(ii) R. A. Fisher

(iii) Cramer and Rao

(iv) None of them

(d) If $T = t(X_1, X_2, ..., X_n)$ is a maximum likelihood estimator of θ and $T(\theta)$ is one-to-one function of θ, then

(i) $T(t)$ is a minimum variance unbiased (MVU) estimator of $T(\theta)$

(ii) $T(t)$ is an unbiased estimator of $T(\theta)$

(iii) $T(t)$ is a maximum likelihood estimator of $T(\theta)$

(iv) All of the above

(e) Factorization theorem for sufficiency is known as

(i) Rao-Blackwell theorem

(ii) Cramer-Rao theorem

(iii) Fisher-Neyman theorem

(iv) None of the above

(f) Minimum chi-square estimators are not necessarily

(i) efficient

(ii) consistent

(iii) unbiased

(iv) All of the above

(g) The estimators by method of moment are

(i) consistent and unbiased

(ii) efficient and sufficient

(iii) consistent but unbiased

(iv) None of the above

(h) Formula for 95% confidence limits for the variance of normal distribution $N(\mu, \sigma^2)$, when μ is unknown is given by

(i) $\frac{ns^2}{\chi^2_{(n-1)(0\cdot025)}} \le \sigma^2 \le \frac{ns^2}{\chi^2_{(n-1)(0\cdot975)}}$

(ii) $\frac{ns^2}{\chi^2_{0\cdot025}} \le \sigma^2 \le \frac{ns^2}{\chi^2_{0\cdot975}}$

(iii) Both *(i)* and *(ii)*

(iv) Neither *(i)* nor *(ii)*

2. Answer the following in brief : 2×8=16

(a) Differentiate between an estimator and an estimate.

(b) What are the different criteria of a good estimator?

(c) Describe unbiased estimator and consistent estimator.

(d) State the invariance property of maximum likelihood estimation.

(e) State Cramer-Rao inequality and mention its uses.

(f) Explain the method of obtaining confidence limits.

(g) Distinguish between point estimation and interval estimation.

(h) Define confidence interval and confidence coefficient.

3. (a) (i) Let $x_1, x_2, ..., x_n$ be a random sample of size n from a Poisson population with parameter λ. Find unbiased estimators of λ and λ^2. Also find the variance of the estimator λ. 5

(ii) Explain minimum variance unbiased estimator (MVUE) with examples. If t is an unbiased estimator of θ, find Cramer-Rao bound of it. 5

Or

(b) *(i)* Define efficiency of an estimator. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Consider two estimates of μ as $T_1 = \frac{1}{n}\overset{n}{\Sigma} x$ and $T_2 = \frac{1}{n+1}\overset{n}{\Sigma} x$. Find relative efficiency of T_2 compared to T_1. 5

(ii) Let $x_1, x_2, \ldots, x_n$ be a random sample of n observations from population having p.d.f. $f(x) = \theta x^{\theta - 1}$, $0 \le x \le 1$, $\theta > 0$. Find a sufficient statistic for θ. 5

4\. *(a)* *(i)* Prove that the sample mean and the sample median are both consistent estimators for the mean of a normal population $N(\mu, \sigma^2)$. 5

(ii) Let $(X_1, X_2, ..., X_n)$ be a random sample from a normal population $N(\mu, 1)$. Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$. 5

Or

(b) *(i)* State the sufficient condition for consistency. Give the statements of the factorization theorem and Rao-Blackwell theorem. 1+4=5

(ii) Let $x_1, x_2, ..., x_n$ be a random sample from an $N(\mu, \sigma^2)$ population. Find sufficient estimators for μ and σ^2. 5

5. Discuss two optimal properties of maximum likelihood method of estimation. 5

6. *(a)* *(i)* State and explain the principle of maximum likelihood estimation of population parameter. 5

(ii) Let $x_1, x_2, ..., x_n$ be a random sample of n observations from $N(\mu, \theta^2)$. Find the maximum likelihood estimators of μ and σ^2. 5

Or

(b) (i) Explain the method of moments in the theory of estimation. 5

(ii) Let $x_1, x_2, ..., x_n$ be a random sample of n observations from a Poisson population whose p.d.f. is $P(x) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, 2, ...$ Find the estimation of θ by the method of moment. 5

7. (a) Explain the method of least squares in the theory of estimation and write the assumptions for estimation. 4+4=8

Or

(b) Write short notes on the following : 4×2=8

(i) Method of minimum variance

(ii) Method of minimum chi-square

8. Explain the confidence intervals and confidence limits. 5

9. (a) Define interval estimation. Let $x_1, x_2, ..., x_n$ be a random sample of n observations from the population having p.d.f. $f(x) = \frac{1}{\theta}$, $0 < x < \theta$. Find $100(1-\alpha)\%$ confidence interval for θ. 8

Or

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with density function $f(x; \theta) = \theta \cdot e^{-\theta x}$, $0 \le x \le \infty$. Find $100(1-\alpha)\%$ (when $0 < \alpha < 1$) confidence interval for the mean of this population, for large sample. 8

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