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3 SEM TDC STS M 1 (N/O)

2017

(November)

STATISTICS

(Major)

Course : 301

(Probability and Distribution—I)

(New Course)

Full Marks : 80
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×8=8

(a) The probability of the intersection of two mutually exclusive events is always

(i) infinity

(ii) zero

(iii) one

(iv) None of the above

(b) If two events A and B are such that $B \subset A$, the relation between $P(A)$ and $P(B)$ is

(i) $P(A) \leq P(B)$

(ii) $P(A) \geq P(B)$

(iii) $P(A) = P(B)$

(iv) None of the above

(c) In tossing three coins at a time, the probability of getting at most one head is

(i) $\frac{3}{8}$

(ii) $\frac{7}{8}$

(iii) $\frac{1}{2}$

(iv) $\frac{1}{8}$

(d) If X is a random variable having its probability density function $f(x)$, then $E(X)$ is called

(i) arithmetic mean

(ii) geometric mean

(iii) harmonic mean

(iv) first quartile

(e) The outcomes of tossing a coin three times are a variable of the type

(i) continuous random variable

(ii) discrete random variable

(iii) discrete as well as continuous random variable

(iv) Neither discrete nor continuous random variable

(f) If X is a random variable with its mean $\overline{X}$, the expression $E(X-\overline{X})^2$ represents

(i) variance of X

(ii) second central moment

(iii) Both *(i)* and *(ii)*

(iv) None of *(i)* and *(ii)*

(g) The correlation coefficient between two independent random variables X and Y is always

(i) 0

(ii) 1

(iii) −1

(iv) None of the above

(h) In case of perfect correlation, both the lines of regression are

(i) coincident

(ii) parallel

(iii) perpendicular to each other

(iv) None of the above

2. Answer the following in brief : 2×8=16

(a) Explain briefly the axiomatic approach to probability.

(b) Two fair dice are thrown simultaneously. What is the probability that the sum of numbers on the dice exceeds 8?

(c) If $P(A \cap B) = \frac{1}{2}$, $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$ and $P(A) = P(B) = P$, find the value of P.

(d) What are the differences between probability mass function and probability density function?

(e) Mention the properties of distribution function.

(f) If a random variable X takes the values 1, 2, 3 with probability $P(X = r) = \frac{r}{6}$; $r = 1, 2, 3$, find $E(X)$ and $V(X)$.

(g) Explain the notion of the joint distribution of two random variables.

(h) What do you understand by correlation between two variables?

3. (a) State and prove the multiplication law of probability for two events A and B. 3

(b) Two dice are thrown once. Find the probability of getting an odd number on the first die or total of 7. 4

4. (a) When are two events said to be (i) independent and (ii) mutually exclusive? Can two events be mutually exclusive and independent at the same time? 4

Or

(b) From 6 positive numbers and 8 negative numbers, 4 numbers are chosen at random and then multiplied. What is the probability that the product is positive? 4

5. (a) Define (i) regression and (ii) line of regression. Why are there two such lines? 4

(b) What do you mean by conditional expectation? Mention some properties of it. 4

Or

(c) The correlation coefficient between two variables X and Y is 0·32. Their covariance is 7·86. The variance of X is 10. Find the standard deviation of Y. 4

6. (a) The joint probability density function of two-dimensional random variable (X, Y) is given by

$$f(x, y) = 2; \; 0 < x < 1, \; 0 < y < x$$
$$= 0, \text{ elsewhere}$$

(i) Find the marginal density function of X and Y.

(ii) Find the conditional density function of Y given $X = x$ and conditional density function of X given $Y = y$.

(iii) Check for independence of X and Y. 6

Or

(b) X and Y are two random variables having the joint density function $f(x, y) = \frac{1}{27}(2x + y)$, where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X = x$. 6

7. A random variable X has the following probability function for various values of x :

x	:	0	1	2	3	4	5	6	7
$P(x)$	:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(a) Find k.

(b) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(3 < X \leq 6)$.

Also find the probability distribution and the distribution function of X. 7

8. (a) Define moment-generating function (m.g.f.). Show that

$$\mu_r' = \left| \frac{d^r}{dt^r} M_x(t) \right|_{t=0}$$ 4

(b) Define cumulant-generating function. Show that

$$k_r\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} k_r(X_i)$$

4

(c) Define probability-generating function (p.g.f.). Find the p.g.f. of *(i)* $X+1$ and *(ii)* $2X$. 4

9. *(a)* Find the moment-generating function for the probability mass function $P(X=x) = {}^nC_x\, P^x q^{n-x};\quad x = 0, 1, 2, \cdots, n$ and hence find mean and variance. 6

Or

(b) For cumulant-generating function (c.g.f.) $k_x(t) = \log_e M_x(t)$, show that

$$\text{Mean} = k_1$$
$$\mu_2 = k_2 = \text{variance}$$
$$\mu_3 = k_3$$
$$\mu_4 = k_4 + 3k_2^2$$

6

10. *(a)* Define mathematical expectation of a discrete and a continuous random variable.

(b) A continuous random variable X has a probability density function

$$f(x) = kx^2 e^{-x};\ x > 0$$

Find k, $E(X)$ and $V(X)$. 2+4=6

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×8=8

(a) The probability of the intersection of two mutually exclusive events is always

(i) ∞

(ii) 0

(iii) 1

(iv) None of the above

(b) If two events A and B are such that $B \subset A$, the relation between $P(A)$ and $P(B)$ is

(i) $P(A) \leq P(B)$

(ii) $P(A) \geq P(B)$

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(d) If X is a random variable having its probability density function $f(x)$, then $E(X)$ is called

(i) arithmetic mean

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(e) The outcomes of tossing a coin three times are a variable of the type

(i) continuous random variable

(ii) discrete random variable

(iii) discrete as well as continuous random variable

(iv) Neither discrete nor continuous random variable

(f) If X is random variable which can take only non-negative values, then

(i) $E(X^2) = [E(X)]^2$

(ii) $E(X^2) \geq [E(X)]^2$

(iii) $E(X^2) \leq [E(X)]^2$

(iv) None of the above

(g) The correlation coefficient between two independent random variables X and Y is always

(i) 0

(ii) 1

(iii) −1

(iv) None of the above

(h) In case of perfect correlation, both the lines of regression are

(i) coincident

(ii) parallel

(iii) perpendicular to each other

(iv) None of the above

2. Answer the following in brief : 2×8=16

(a) Write down the axiomatic definition of probability.

(b) In an experiment, a coin is thrown five times. Write down the sample space. How many sample points are there in sample space?

(c) If A and B are two independent events, prove that A and $\overline{B}$ are also independent.

(d) Two fair dice are thrown simultaneously. What is the probability that the sum of numbers on the dice exceeds 8?

(e) If a random variable X takes the values 1, 2, 3 with probability $P(X = r) = \frac{r}{6}$; $r = 1, 2, 3$, find $E(X)$ and $V(X)$.

(f) What are probability mass function and probability density function?

(g) Define correlation coefficient.

(h) What do you mean by marginal probability function?

3. A random variable X has the following probability function for various values of x:

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(a) Find k.

(b) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(3 < X \leq 6)$.

Also find the probability distribution and the distribution function of X. 7

4. *(a)* Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. What is the chance that exactly two of them will be children? 3

(b) State and prove the multiplication law of probability for two events A and B. 3

(c) Two dice are thrown once. Find the probability of getting an odd number on the first die or total of 7. 4

5. *(a)* State and prove Bayes' theorem. 5

Or

(b) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output, 5%, 4% and 2% are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? 5

6. (a) If A and B are two independent events, then show that $\overline{A}$ and $\overline{B}$ are also independent. 2

(b) A and B are events, such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$. Find (i) $P(B)$ and (ii) $P(A \cap \overline{B})$. 2

7. (a) Define (i) regression and (ii) line of regression. Why are there two such lines? 4

(b) What do you mean by conditional expectation? State some properties of it. 4

Or

(c) The correlation coefficient between two variables X and Y is 0·32. Their covariance is 7·86. The variance of X is 10. Find the standard deviation of Y. 4

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(ii) Find the conditional density function of Y given $X = x$ and conditional density function of X given $Y = y$.

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Or

(b) X and Y are two random variables having the joint density function $f(x, y) = \frac{1}{27}(2x + y)$, where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X = x$. 6

9. *(a)* Define mathematical expectation of a discrete and a continuous random variable. 2

(b) A continuous random variable X has a probability density function

$$f(x) = kx^2 e^{-x};\ x > 0$$

Find k, $E(X)$ and $V(X)$. 4

10. *(a)* Define moment-generating function (m.g.f.). Show that

$$\mu'_r = \left| \frac{d^r}{dt^r} M_x(t) \right|_{t=0}$$

4

Or

(b) Define probability, generating function (p.g.f.). Find the p.g.f. of *(i)* $X+1$ and *(ii)* $2X$. 4

11. For cumulant-generating function (c.g.f.) $k_x(t) = \log_e M_x(t)$, show that

$$\text{Mean} = k_1$$
$$\mu_2 = k_2 = \text{variance}$$
$$\mu_3 = k_3$$
$$\mu_4 = k_4 + 3k_2^2$$

6

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