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3 SEM TDC STS M 2 (N/O)

2017

(November)

STATISTICS

(Major)

Course : 302

(Numerical Methods)

(New Course)

Full Marks : 80
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) The differences of constant function are

(i) zero, i.e., $\Delta c = 0$

(ii) 1, i.e., $\Delta c = 1$

(iii) –1, i.e., $\Delta c = -1$

(iv) None of the above

(b) If $f(x)$ be a polynomial of nth degree, then

(i) $\Delta^n f(x) = 0$

(ii) $\Delta^{n-1} f(x) =$ constant

(iii) $\Delta^{n+1} f(x) = 0$

(iv) $\Delta^{n+1} f(x) =$ constant

(c) The value of Δe^x with interval of differencing h is

(i) $e^x - e^{x+h}$

(ii) $e^x - e^{x-h}$

(iii) $e^{x+h} + e^x$

(iv) $e^{x+h} - e^x$

(d) Newton's divided difference formula for interpolation is used for

(i) equal interval

(ii) unequal interval

(iii) central difference

(iv) None of the above

(e) Stirling's interpolation formula is the average of

(i) Newton's forward formula and Newton's backward formula

(ii) Bessel's interpolation formula and Laplace-Everett interpolation formula

(iii) Gauss's forward formula and Gauss's backward formula

(iv) None of the above

(f) To find the interpolated value near the centre of the table, which one of the formulae should be used?

(i) Gauss's backward formula

(ii) Stirling's formula

(iii) Lagrange's formula

(iv) Newton's forward formula

(g) Simpson's one-third rule is called

(i) straight line formula

(ii) hyperbolic formula

(iii) parabolic formula

(iv) None of the above

(h) Transcendental equation can be solved by using

(i) bisection method only

(ii) iterative method only

(iii) Newton-Raphson method only

(iv) All of the above

2. (a) Discuss if operators E and Δ obey the distributive, commutative, associative and indices of laws of algebra. 6

(b) Evaluate

$$\frac{\Delta^2}{E}x^3$$

3

3. (a) Use the method of finite differences to sum the series $1^3+2^3+3^3+...+n^3$. 6

Or

(b) Sum to n terms the series whose xth term is

(i) $x(x-1)(x-2)$

Or

(ii) $\frac{1}{(x+1)(x+2)(x+3)}$ 6

4. Answer any *three* of the following :

(a) (i) Establish an interpolation formula for equal intervals.

(ii) From the following data, find the value of U_{47}:

$$U_{46} = 0 \cdot 2884,\ U_{48} = 0 \cdot 5356,$$

$$U_{49} = 0 \cdot 6513,\ U_{50} = 0 \cdot 7620$$

5+5=10

(b) What are divided differences? Prove that divided differences of a polynomial of nth degree is constant. 5+5=10

(c) (i) Deduce Lagrange's formula for interpolation.

(ii) Find the form of the function, given—

x	0	1	2	5
$f(x)$	2	3	12	147

5+5=10

(d) (i) Derive Gauss's forward formula for central differences.

(ii) Use Stirling's formula to find $f(35)$, given—

$f(20) = 512,\ f(30) = 439,\ f(40) = 346$ and $f(53) = 243$. 5+5=10

5. (a) What is numerical differentiation?

(b) Given the following pairs of values of x and $y = f(x)$:

x	1	2	4	8	10
$y = f(x)$	0	1	5	21	27

Determine numerically the first derivative of $f(x)$ at $x = 4$. 3+6=9

6. Answer any *two* of the following :

(a) (i) State and prove Simpson's $\frac{3}{8}$th rule of numerical integration.

(ii) Calculate the approximate value of $\int_0^6 \frac{dx}{(1+x)^2}$ using Simpson's $\frac{1}{3}$rd rule.

5+4=9

(b) (i) Describe Newton-Raphson method. In which situation this method is applicable?

(ii) By using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$ which is nearer to $x = 2$, correct to three places of decimals. 5+4=9

(c) (i) Describe bisection method. In which situation the bisection method is applicable?

(ii) Solve $x^3 - x - 1 = 0$ for a root between 1 and 2 by bisection method (do three iterations only).

5+4=9

(Old Course)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) If the nth differences of a tabulated function $f(x)$ are constant, the value of independent variables are taken at equal intervals, then

(i) $f(x)$ is a polynomial of degree n

(ii) $f(x)$ is constant

(iii) $f(x)$ is zero

(iv) $f(x)$ is a polynomial of degree $n-1$

(b) The nth backward difference is given by

(i) $\Delta[\nabla^{n-1} f(x)]$

(ii) $\nabla[\nabla^{n-1} f(x)]$

(iii) $\nabla[\Delta^{n-1} f(x)]$

(iv) $\nabla^{n-1} f(x) - \nabla^{n-1} f(x-h)$

(c) Which of the following is not correct?

(i) $E^m E^n f(x) = E^{m+n} f(x)$

(ii) $E\nabla \equiv \nabla E \equiv \Delta$

(iii) $E^{-n} f(x) = f(x - nh)$

(iv) $E^2 f(x) = [Ef(x)]^2$

(d) If $y_0 = 580$, $y_1 = 556$, $y_2 = 520$, $y_4 = 384$, then the value of y_3 is

(i) 1860

(ii) 930

(iii) 465

(iv) 234

(e) Which of the following statements is incorrect?

(i) The nth divided difference can be expressed as the quotient of two determinants each of order $n+1$.

(ii) The nth divided differences of a polynomial of the nth degree are constant.

(iii) The nth divided difference of $a_n x^n$ is a_n.

(iv) The nth divided difference of x^n is zero.

(f) Which of the following relations is valid?

(i) $E^{\frac{1}{2}} = \frac{1}{2}\mu + \delta$

(ii) $E^2 = \mu + \frac{1}{2}\delta$

(iii) $E^{\frac{1}{2}} = \mu + \frac{1}{2}\delta$

(iv) $E = \mu + \delta$

(g) The trapezoidal rule is applicable to the polynomial of the _____ or _____.

(i) second degree, curve

(ii) first degree, circle

(iii) first degree, straight line

(iv) nth degree, ellipse

(h) An algebraic equation of degree n, where n is a positive integer, has

(i) n roots

(ii) n^2 roots

(iii) n^3 roots

(iv) None of the above

2. Answer the following in brief : 2×8=16

(a) Establish the relation, $E\Delta \equiv \Delta E$.

(b) Evaluate

$$\frac{\Delta^2 x^3}{Ex^3}$$

The interval of differencing is unity.

(c) State the Newton-Gregory backward interpolation formula.

(d) What are the advantages arising from the use of central differences in interpolation?

(e) State the Weddle's rule for numerical integration. What are the basic conditions to apply Weddle's rule?

(f) Write a short note on inverse interpolation.

(g) Write two properties of algebraic and transcendental equations.

(h) State the formula of Newton-Raphson method.

3. (a) (i) Define Δ operator and E operator.

(ii) What is the difference between $\left(\frac{\Delta u_x}{Eu_x}\right)^2$ and $\left(\frac{\Delta^2 u_x}{E^2 u_x}\right)$? If $u_x = x^2$ and interval of differencing is unity, find out the expressions for both.

4+5=9

Or

(b) (i) What are backward differences of a polynomial? State the relationship of backward operator ∇ and E operator.

(ii) Use the method of finite differences sum to n terms whose xth term is $x(x+2)(x+4)$.

4+5=9

4. Answer any *two* of the following :

(a) (i) What do you understand by interpolation? What are the underlying assumptions for the validity of various methods used for interpolation?

(ii) Derive the Newton-Gregory backward interpolation formula for equal intervals.

(iii) If $U_0 = 0$, $U_{10} = 15$, $U_{20} = 50$, estimate U_{18}. 4+5+4=13

(b) (i) What are divided differences? Show that nth divided differences of a polynomial of the nth degree are constant.

(ii) Establish the relation between divided differences and ordinary differences.

(iii) Find the third divided differences with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$. 5+4+4=13

(c) (i) Derive the Gauss's forward interpolation formula for equal intervals.

(ii) Establish the relation

$$\Delta\nabla \equiv \nabla\Delta \equiv \Delta - \nabla = \delta^2$$

(iii) Use Stirling's formula to find y_{28}, given—

$$y_{20} = 49225, \quad y_{25} = 48316,$$
$$y_{30} = 47236, \quad y_{35} = 45926,$$
$$y_{40} = 44306$$

5+4+4=13

(d) *(i)* State Lagrange's interpolation formula and discuss its merits and demerits.

(ii) Apply Lagrange's formula to find $f(5)$, given that $f(1)=2$, $f(2)=4$, $f(3)=8$, $f(4)=16$, $f(7)=128$ and explain why the result differs from 2^5.

(iii) What is inverse interpolation? Name three methods of inverse interpolation and describe one of them. 4+5+4=13

5. Answer any *three* of the following :

(a) What is numerical differentiation? Find first and second derivatives of the function given below at the point $x=1\cdot2$: 2+5=7

x	1	2	3	4	5
y	0	1	5	6	8

(b) State and prove Simpson's $\frac{1}{3}$rd rule for numerical integration. What is the effect of—

(i) change of origin;

(ii) change of scale on this rule? 5+2=7

(c) State the trapezoidal rule of numerical integration. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's rule. 2+5=7

(d) Define algebraic and transcendental equations. Give example of each.

Find the root of $x^4 - x - 10 = 0$ which is nearer to $x = 2$, correct to three places of decimals by using Newton-Raphson method. 2+5=7

(e) Discuss the bisection method for solution of transcendental equations.

Solve $x^3 - x - 1 = 0$ for a root between 1 and 2 by bisection method (do three iterations only). 4+3=7

(f) Write short notes on the following : 3½+3½=7

(i) Regula falsi method

(ii) Iteration method

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