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5 SEM TDC STS M 1 (N & O)

2016

(November)

STATISTICS

(Major)

Course : 501

(Estimation)

(Both New and Old Course)

The figures in the margin indicate full marks for the questions

Full Marks : 80

Pass Marks : 24 (New)/32 (old)

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) If the expected value of an estimator is not equal to its parametric function $r(\theta)$, it is said to be a

(i) unbiased estimator

(ii) biased estimator

(iii) consistent estimator

(iv) None of the above

(b) If $x_1, x_2, \cdots, x_n$ be a random sample drawn from a $N(\mu, \sigma^2)$ population, the sufficient statistics from is

(i) $\Sigma(x_i - \overline{x})$

(ii) Σx_i

(iii) Σx_i^2

(iv) None of the above

(c) Factorisation theorem for sufficiency is known as

(i) Rao-Blackwell theorem

(ii) Cramer-Rao theorem

(iii) Fisher-Neyman theorem

(iv) None of the above

(d) Let θ be an unknown parameter and T_1 be an unbiased estimator of θ. If $\text{var}(T_1) \le \text{var}(T_2)$ for T_2 to be any other unbiased estimator. Then T_1 is known as

(i) minimum variance unbiased estimator

(ii) unbiased and efficient estimator

(iii) consistent and efficient estimator

(iv) unbiased, consistent and minimum variance estimator

(e) Let T_n be an estimator based on a sample $x_1, x_2, \cdots, x_n$ of the parameter θ. Then T is a consistent estimator of θ if

(i) $P(T_n - \theta > \varepsilon) = 0 \;\; \forall \, \varepsilon > 0$

(ii) $P(|T_n - \theta| < \varepsilon) = 0$

(iii) $\lim_{n \to \infty} P(|T_n - \theta| > \varepsilon) = 0 \;\; \forall \, \varepsilon > 0$

(iv) $\lim_{n \to \infty} P(T_n - \theta > \varepsilon) = 0 \;\; \forall \, \varepsilon > 0$

(f) The maximum likelihood estimator which are obtained by maximising the function of joint density of random variables are generally

(i) unbiased and consistent

(ii) unbiased and inconsistent

(iii) consistent and invariant

(iv) invariant and unbiased

(g) The 95% confidence limit for θ in case of large samples and of density function

$$f(x, \theta) = \theta e^{-\theta x}; 0 < x < \infty$$

is

(i) $(1 \pm 1 \cdot 96 / \sqrt{n})\, \bar{x}$

(ii) $(1 \pm 1 \cdot 96\bar{x} / \sqrt{n}) / \bar{x}$

(iii) $(1 \pm 1 \cdot 96 / \sqrt{n}) / \bar{x}$

(iv) None of the above

(h) In case there exist more than one set of confidence interval with the same confidence coefficient, we look for

(i) the shortest of all intervals

(ii) the largest of all intervals

(iii) Both (i) and (ii)

(iv) None of the above

2. Answer the following in brief : 2×8=16

(a) Mention the criterion that should be satisfied by a good estimator.

(b) If T_n is unbiased estimator of θ, then show that ${T_n}^2$ is a biased estimator of θ^2.

(c) State the sufficient condition for consistency.

(d) If T_1 and T_2 are unbiased estimators of θ and $\text{var}(T_1) = 0 \cdot 2$ and $\text{var}(T_2) = 0 \cdot 6$, compute the relative efficiency of T_1 as compared to T_2.

(e) Elucidate what technique you will apply when maximum likelihood estimation fails.

(f) Define confidence interval and confidence coefficient.

(g) State the invariance property of MLE.

(h) Explain the method of obtaining confidence limit.

3. *(a)* *(i)* Let $x_1, x_2, \cdots, x_n$ be a random sample from $N(\mu, \sigma^2)$. Find sufficient estimator for μ and σ^2. 5

(ii) If $x_1, x_2, \cdots, x_n$ is a random sample where variate x takes the value 1 with probability p and takes the value 0 with probability $1-p$, then show that $\bar{x}(1-\bar{x})$ is a consistent estimator of $p(1-p)$. 5

Or

(b) *(i)* Prove that for Cauchy's distribution not sample mean but sample median is a consistent estimator of the population mean. 5

(ii) If T_1 and T_2 be two unbiased estimators of $r(\theta)$ having the same variance and ρ is the correlation coefficient between them, then show that $\rho \geq 2e-1$ where e is the efficiency of each estimator. 5

4. *(a)* *(i)* If T_1 and T_2 be two unbiased estimators of $r(\theta)$ with variances σ_1^2 and σ_2^2 and correlation coefficient ρ. What is the variance of such a combination? 5

(ii) Obtain the minimum variance bound estimator (MVBE) for μ of the normal population $N(\mu, \sigma^2)$ where σ^2 is known. 5

Or

(b) *(i)* Find if MVB estimator exists for θ in the Cauchy's population with density function

$$f(x, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2}; \qquad -\infty < x < \infty$$

5

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample from a uniform population $[0, \theta]$. Find a sufficient estimator for θ. 5

5. State the important properties of MLE. 5

6. *(a)* *(i)* Find the MLE for the parameter λ of the Poisson distribution on the basis of a sample of size n. Also find its variance. 5

(ii) Obtain the most general form of distribution differentiable in θ for which sample mean is MLE. 5

Or

(b) *(i)* Write an explanatory note on the method of minimum variance in the theory of estimation. 5

(ii) Explain the method of moments in the theory of estimation. 5

7. *(a)* In random sampling from normal population $N(\mu, \sigma^2)$, find the MLE for—

(i) μ, when σ^2 is known;

(ii) σ^2, when μ is known;

(iii) the simultaneous estimation of μ and σ^2. 8

Or

(b) State Cramer-Rao inequality for lower bound of variance of an unbiased estimator. Show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$ in random sample from

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta} \quad ; \quad 0 < x < \infty, \quad 0 < \theta < \infty$$

$$= 0 \quad ; \quad \text{otherwise}$$

is an MVB estimator of θ and has variance θ^2 / n. 2+6=8

(Turn Over)

8. Distinguish between point estimation and interval estimation. 5

9. *(a)* Obtain $100(1-\alpha)\%$ confidence interval for the parameter μ and σ^2 of the normal distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < x < \infty$$
$$-\infty < \mu < \infty$$
$$\sigma > 0$$ 8

Or

(b) Write an explanatory note on interval estimation. A random sample of size 100 has mean 15, the population variance 25. Find the interval estimate of population mean with a confidence level of 95% and 99%. 4+4=8

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