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5 SEM TDC STS M 2 (N/O)

2016

(November)

STATISTICS

(Major)

Course : 502

(Testing of Hypotheses)

(New Course)

Full Marks : 80
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternatives from the following : 1×8=8

(a) A sequence of symbols shows lack of randomness if there are

(i) too many runs

(ii) too few runs

(iii) Both *(i)* and *(ii)*

(iv) Neither *(i)* nor *(ii)*

(b) Whether a test is one-sided or two-sided, it depends on

(i) alternative hypothesis

(ii) composite hypothesis

(iii) null hypothesis

(iv) simple hypothesis

(c) The ratio of the likelihood function under H_0 and under the entire parametric space is called

(i) probability ratio

(ii) sequential probability ratio

(iii) likelihood ratio

(iv) None of the above

(d) The power of a test is related to

(i) type-I error

(ii) type-II error

(iii) type-I and type-II errors both

(iv) None of the above

(e) To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$, when the population SD is known, the appropriate test is

(i) *t*-test

(ii) *z*-test

(iii) chi-square test

(iv) None of the above

(f) Student's t-test was initiated by

(i) R. A. Fisher

(ii) G. W. Snedecor

(iii) W. S. Gosset

(iv) W. G. Cochran

(g) Ordinary sign test considers the difference of observed values from the hypothetical median value in terms of

(i) signs only

(ii) magnitudes only

(iii) sign and magnitude both

(iv) None of the above

(h) If in Wilcoxon's signed rank test the sample size is large, the statistic T^+ is distributed with mean

(i) $\frac{n(n+1)}{4}$

(ii) $\frac{n(n+1)}{2}$

(iii) $\frac{n(2n+1)}{4}$

(iv) $\frac{n(n-1)}{4}$

2. Answer the following in brief : 2×8=16

(a) Distinguish between simple and composite hypothesis with examples.

(b) Describe likelihood ratio test.

(c) What are the advantages of non-parametric test?

(d) Define *t*-statistic for comparing the means of two normal populations.

(e) Explain the concept of the best critical region.

(f) What are the basic steps involved in any non-parametric test of hypothesis?

(g) What do you understand by the test of goodness of fit?

(h) When do you use paired *t*-test?

3. *(a)* Define type-I, type-II errors and power. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x, \theta) = \theta e^{-\theta x}$, $0 \leq x \leq \infty$, obtain the values of type-I and type-II errors. Also obtain the power function of the test. 8

(b) A random sample of size n is drawn from a normal population $N(\mu, \sigma^2)$. Assuming that σ^2 is known, find the best critical region for testing $H_0 : \mu = 100$ against $H_1 : \mu = 120$ and find the power of the test. 6

4. (a) Let $x_1, x_2, \cdots, x_n$ be a random sample from a normal population with mean μ and variance σ^2; μ and σ^2 being unknown. Test $H_0 : \mu = \mu_0$ (specified) against $H_1 : \mu \neq \mu_0$, $0 < \sigma^2 < \infty$. Show that the likelihood ratio test is same as the two-tailed t-test. 7

Or

(b) Show that for the normal distribution with zero mean and variance σ^2, the best critical region for $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1$ is of the form

$$\sum_{i=1}^{n} x_i^2 \leq a_\alpha \text{ for } \sigma_0 > \sigma_1$$

$$\sum_{i=1}^{n} x_i^2 \geq b_\alpha \text{ for } \sigma_0 < \sigma_1$$ 7

5. Describe Kolmogorov-Smirnov test of goodness of fit in case of one sample. 6

6. *(a)* What is non-parametric test? What are the assumptions associated with non-parametric test? 4

(b) Describe the sign test, stating the assumptions and the hypothesis of the test. 7

Or

(c) Describe the run test for one sample problem, stating clearly the hypothesis to be tested. 7

7. There are two populations and P_1 and P_2 are the proportions of members in the two populations belonging to low-income group. It is desired to test the hypothesis $H_0 : P_1 = P_2$. Explain clearly the procedure that you would follow to carry out the test at 5% level of significance.

State the theorem on which the above test is based. 8

8. Describe in detail how you would test the equality of population variances with the help of F-test. 6

9. Describe the following test of significance : 4

(a) Large sample test for single mean

Or

(b) t-test for an observed sample correlation coefficient

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternatives from the following : 1×8=8

(a) A sequence of symbols shows lack of randomness if there are

(i) too many runs

(ii) too few runs

(iii) Both *(i)* and *(ii)*

(iv) Neither *(i)* nor *(ii)*

(b) Whether a test is one-sided or two-sided, it depends on

(i) alternative hypothesis

(ii) composite hypothesis

(iii) null hypothesis

(iv) simple hypothesis

(c) The ratio of the likelihood function under H_0 and under the entire parametric space is called

(i) probability ratio

(ii) sequential probability ratio

(iii) likelihood ratio

(iv) None of the above

(d) The power of a test is related to

(i) type-I error

(ii) type-II error

(iii) type-I and type-II errors both

(iv) None of the above

(e) To test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$, when the population SD is known, the appropriate test is

(i) t-test

(ii) z-test

(iii) chi-square test

(iv) None of the above

(f) Student's t-test was initiated by

(i) R. A. Fisher

(ii) G. W. Snedecor

(iii) W. S. Gosset

(iv) W. G. Cochran

(Continued)

(g) Ordinary sign test considers the difference of observed values from the hypothetical median value in terms of

(i) signs only

(ii) magnitudes only

(iii) sign and magnitude both

(iv) None of the above

(h) Sequential probability ratio test (SPRT) was initiated by

(i) R. A. Fisher

(ii) A. Wald

(iii) G. W. Snedecor

(iv) Thomas Bayes

2. Answer the following in brief : 2×8=16

(a) Distinguish between simple and composite hypothesis with examples.

(b) Describe likelihood ratio test.

(c) What are the advantages of non-parametric test?

(d) Define *t*-statistic for comparing the means of two normal populations.

(e) Explain the concept of the best critical region.

(f) What are the basic steps involved in any non-parametric test of hypothesis?

(g) What do you understand by the test of goodness of fit?

(h) When do you use paired t-test?

3. Define type-I, type-II errors and power. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x, \theta) = \theta e^{-\theta x}$, $0 \leq x \leq \infty$, obtain the values of type-I and type-II errors. Also obtain the power function of the test. 8

4. (a) State Neyman-Pearson lemma. 2

(b) A random sample of size n is drawn from a normal population $N(\mu, \sigma^2)$. Assuming that σ^2 is known, find the best critical region for testing $H_0 : \mu = 100$ against $H_1 : \mu = 120$ and find the power of the test. 6

Or

(c) Show that for the normal distribution with zero mean and variance σ^2, the best critical region for $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1$ is of the form

$$\sum_{i=1}^{n} x_i^2 \leq a_\alpha \text{ for } \sigma_0 > \sigma_1$$

$$\sum_{i=1}^{n} x_i^2 \geq b_\alpha \text{ for } \sigma_0 < \sigma_1$$ 6

5. There are two populations and P_1 and P_2 are the proportions of members in the two populations belonging to low-income group. It is desired to test the hypothesis $H_0: P_1 = P_2$. Explain clearly the procedure that you would follow to carry out the test at 5% level of significance.

   State the theorem on which the above test is based. 8

6. *(a)* Describe in detail how you would test the equality of population variances with the help of *F*-test. 6

   Or

   (b) Describe chi-square test of independence of attributes. 6

7. *(a)* Describe the sign test, stating the assumptions and the hypothesis of the test. 7

   Or

   (b) Describe the run test for one sample problem, stating clearly the hypothesis to be tested. 7

8. *(a)* Give in brief the idea of sequential probability ratio test (SPRT). 5

(b) Define the OC function and ASN function in sequential analysis. Derive their approximate expressions for the sequential probability ratio test of a simple hypothesis against a simple alternative. 7

(c) Develop SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$, where θ is the parameter of a Poisson distribution. Write down the approximate expressions for OC and ASN functions of the test. 7

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