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3 SEM TDC STS M 2 (N/O)

2016

(November)

STATISTICS

(Major)

Course : 302

(Numerical Methods)

(New Course)

Full Marks : 80
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) Which one of the following is not true?

(i) $\nabla y_n = y_{n-1} - y_n$

(ii) $\Delta^2 y_0 = y_2 - 2y_1 + y_0$

(iii) $\Delta y_1 = y_2 - y_1$

(iv) $\nabla^2 y_2 = y_2 - 2y_1 + y_0$

(b) Divided differences are symmetric functions of

(i) arguments

(ii) entries

(iii) both arguments and entries

(iv) neither arguments nor entries

(c) The central difference operator δ is given by

(i) $E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

(ii) $E^{\frac{1}{2}} + E^{-\frac{1}{2}}$

(iii) $E^{-\frac{1}{2}} - E^{\frac{1}{2}}$

(iv) $\frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})$

(d) Inverse interpolation method can be used

(i) only when arguments are at equal interval

(ii) for both equal and unequal intervals of arguments

(iii) only when entries are at equal interval

(iv) only when both arguments and entries are at unequal interval *(e)* Gauss forward formula is suitable for interpolation

(i) near the beginning of a series

(ii) near the middle of a series

(iii) of both beginning and end of a series

(iv) near the end of a series

(f) Weddle's rule can be applied when the number of subintervals is

(i) multiple of 4

(ii) multiple of 6

(iii) multiple of 8

(iv) any positive integer

(g) Which one is not true?

(i) Zero of a polynomial $f(x)$ is given by $f(x) = 0$

(ii) Every polynomial of degree n has exactly n roots

(iii) Every polynomial of degree n has exactly n real roots

(iv) $(x^n - 1)$ is a polynomial of degree n in x

(h) Which one is not true?

(i) Bisection method always converges

(ii) Newton-Raphson method is for finding a real root of an equation

(iii) Regula falsi method cannot be used for transcendental equation

(iv) Newton-Raphson method does not always converge

2. *(a)* Define the operators E, ∇ and Δ, and establish that *(i)* $\Delta = \nabla E$ and *(ii)* $E = 1 + \Delta = e^{hD}$, where h = interval of differencing and $Dy_x = \frac{d}{dx} y_x$. 6

(b) Prove that

$$e^x = \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$$

the interval of differencing being h. 3

3. *(a)* Using the method of separation of symbols, show that

$$e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!} u_0 + \cdots\right) = u_0 + u_1 x + u_2 \frac{x^2}{2!} + \cdots$$ 6

Or

(*b*) Use the method of finite differences to find the sum up to n terms of series

(*i*) $\sum \frac{x+3}{x(x+1)(x+2)}$

Or

(*ii*) $1^3 + 2^3 + 3^3 + \cdots$ 6

4. Answer any *three* of the following :

(*a*) Derive Newton's backward interpolation formula.

Given :

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

Find the form of the function $f(x)$.

5+5=10

(*b*) For interpolation with equal interval, when do we use—

(*i*) Newton's formulae;

(*ii*) Gauss' formulae?

Write down the Stirling's central difference formula and mention in what situation it is more suitable.

Given, $f(20) = 49225$, $f(25) = 48316$, $f(30) = 47236$, $f(35) = 45926$, $f(40) = 44306$. Use Stirling's formula to find $f(28)$. 5+5=10

(c) Write down Newton's interpolation formula with divided difference. Given, $f(0) = 8$, $f(1) = 68$, $f(5) = 123$. Find $f(2)$.

Describe a method for inverse interpolation and give two applications of this method. 5+5=10

(d) Prove that the nth divided differences of a polynomial of degree n are constants.

If $f(0) = 0$, $f(1) = 1$, $f(2) = 20$, find the value of x for which $f(x) = 19$. 6+4=10

5. *(a)* Write a note on numerical differentiation.

(b) What is numerical quadrature?

(c) Define algebraic and transcendental equations. Give example of each.

4+2+3=9

6. Answer any *two* of the following :

(a) Using the relation

$$1-\nabla = e^{-hD}$$

or, otherwise, prove that

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

where h = interval of differencing and $Dy_x = \frac{d}{dx} y_x$.

Given :

x	1·0	1·2	1·4	1·6	1·8	2·0
$f(x)$	0	·128	·544	1·296	2·432	4

Find $f'(2)$. 5+4=9

(b) Derive Simpson's one-third rule. If $u_x = a + bx + cx^2$, prove that

$$\int_1^3 u_x dx = 2u_2 + \frac{1}{12}(u_0 - 2u_2 + u_4)$$ 5+4=9

(c) Describe Newton-Raphson method for finding real root of an equation. Mention one of the limitations of the method.

Solve $x^3 - x - 1 = 0$ for a root between 1 and 2 by bisection method (do three iterations only). 5+4=9

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) If $\phi_n(x)$ is a polynomial of nth degree, $\Delta^n \phi_n(x)$ will be

(i) a constant

(ii) zero

(iii) a function of x

(iv) None of the above

(b) With usual notation, nth forward difference is given by

(i) $\Delta^n f(x) = \Delta^{n-1} f(x+h) - \Delta^{n-1} f(x)$

(ii) $\Delta^n f(x) = \Delta^{n-1} f(x) - \Delta^{n-1} f(x+h)$

(iii) $\Delta^n f(x) = \Delta^n f(x+h) - \Delta^{n-1} f(x)$

(iv) $\Delta^n f(x) = \Delta^{n+1} f(x+h) - \Delta^n f(x)$

(c) The process of computing the value of the function outside the range of given values of the variable is called

(i) interpolation

(ii) extrapolation

(iii) divided difference

(iv) numerical differentiation

(d) Which of the following interpolation formulae can be used for inverse interpolation?

(i) Newton's forward interpolation formula

(ii) Newton's backward interpolation formula

(iii) Lagrange's interpolation formula

(iv) Newton's divided difference formula

(e) Bessel's interpolation formula is the most appropriate to estimate for a value in a series which lies

(i) at the end

(ii) in the beginning

(iii) in the middle of the central interval

(iv) outside the series

(f) The process by which the derivatives of a function at some values of the independent variable are found out for given set of values of the function is known as

(i) numerical integration

(ii) numerical differentiation

(iii) Newton-Raphson method

(iv) All of the above

(g) To apply trapezoidal rule of numerical integration, the number of subintervals must be

(i) an even positive integer

(ii) multiple of 3

(iii) any positive number

(iv) None of the above

(h) Transcendental equations can be solved by

(i) bisection method

(ii) iterative method

(iii) Newton-Raphson method

(iv) All of the above

2. Answer the following in brief : 2×8=16

(a) Establish the relation, $(1+\Delta)(1-\nabla)\equiv 1$.

(b) Evaluate

$$\left(\frac{\Delta^2}{E}\right)x^3$$

the interval of differencing being unity.

(c) What is interpolation? What are the underlying assumptions for the validity of various methods of interpolation?

(d) Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x)=x^3-2x$.

(e) What is the difference between Lagrange's method and Newton's method as a method for interpolation?

(f) What is inverse interpolation? Name three methods which are generally used for inverse interpolation.

(g) What are the basic conditions to apply in Simpson's one-third rule?

(h) What are meant by algebraic and transcendental equations?

3. (a) (i) Define the terms arguments entry, leading term and interval of differencing in a difference table. 4

(ii) If Δ and ∇ be the first descending difference operator and first ascending difference operator respectively of a function $f(x)$, show that $(\Delta - \nabla) \equiv \Delta\nabla$. 5

Or

(b) What do you mean by calculus of finite differences? Prove that the nth differences of a rational integral function (polynomial) of the nth degree are constant when the values of the independent variable are at equal intervals.

2+7=9

4. Answer any *two* of the following :

(a) (i) Prove that

$$f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$$

(ii) Use the method of separation of symbols to prove the following identity :

$$u_0 + u_1 + u_2 + \cdots + u_n = {}^{n+1}C_1 u_0 + {}^{n+1}C_2 \Delta u_0 + {}^{n+1}C_3 \Delta^2 u_0 + \cdots + \Delta^n u_0$$

(iii) Derive the Newton-Gregory forward interpolation formula for equal intervals. 4+4+5=13

(b) What are the divided differences? Derive the Newton's divided difference formula. Establish the relation between divided differences and ordinary differences. Prove that

$$\underset{bcd}{\Delta}^3 (1/a) = -\frac{1}{abcd}$$

where Δ is the divided difference operator. 1+4+4+4=13

(c) Define central difference operator δ and the average operator μ. Derive Gauss' backward interpolation formula. Apply a central difference formula to obtain $f(25)$. Given that $f(20) = 14$, $f(24) = 32$, $f(28) = 35$, $f(32) = 40$. 2+5+6=13

(d) Derive Lagrange's interpolation formula. In which situations normally Lagrange's interpolation formula is used? Using this formula, prove that

$$u_1 = u_3 - 0 \cdot 3(u_5 - u_{-3}) + 0 \cdot 2(u_{-3} - u_{-5})$$

6+2+5=13

5. Answer any *three* of the following :

(a) Find the first two derivatives of $f(x)$ at $x = 1$ from the following table : 7

x	-2	-1	0	1	2	3	4
$f(x)$	104	17	0	-1	8	69	272

(b) What is numerical quadrature? State the general Gauss-Legendre quadrature formula for equidistant ordinates. Also derive trapezoidal rule using this quadrature formula. 1+1+5=7

(c) Evaluate the approximate value of the integral

$$\int_0^1 \sqrt{8 + x + x^2}\, dx$$

using Simpson's one-third rule (correct to six decimals, using 11 ordinates). 7

(d) Discuss the regula falsi method for solution of transcendental equations. Use the method to find the real root of the equation $x^6 - x^4 - x^3 - 1 = 0$ which lies between 1 and 2. 4+3=7

(e) Describe Newton-Raphson method. In which situation this method is applicable? 5+2=7

(f) In what situation the bisection method is applicable? Describe the method and mention clearly how the percentage error is used to determine the iteration. 1+6=7

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