Total No. of Printed Pages—7

6 SEM TDC MTH M 1

2019

(May)

MATHEMATICS

(Major)

Course : 601

(A : Metric Spaces and B : Statistics)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Metric Spaces

(*Marks* : 40)

1. *(a)* Define neighbourhood of a point. 1

(b) Let (X, d) be a metric space and $A \subseteq X$. Show that A is open $\Leftrightarrow A^\circ = A$, where $A^\circ =$ interior of A. 2

(c) If d and d^* are metrices on a non-empty set X, then prove that $d + d^*$ is also a metric on X. 3

2. (a) Prove that in a metric space (X, d), any arbitrary intersection of closed sets is closed. 4

Or

Prove that in a metric space (X, d), every open sphere is an open set.

(b) Define diameter of a set. 1

3. Let (X, d) be a metric space and $A \subseteq X$. Show that A is closed if and only if A contains all its limit points. 4

Or

Let (Y, d_Y) be a subspace of a metric space (X, d) and $A \subseteq Y$. Show that the closure of A in Y is $\overline{A} \cap Y$, where $\overline{A}$ is the closure of A in X.

4. (a) Define a complete metric space. 1

(b) Prove that in a metric space, every convergent sequence is a Cauchy sequence. 2

(c) Let (X, d) be a metric space. If $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $x_n \to x$, $y_n \to y$, prove that

$$d(x_n, y_n) \to d(x, y)$$ 4

(d) Let (X, d) be a complete metric space and $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \to 0$. Then show that the intersection $\bigcap_{n=1}^{\infty} F_n$ contains exactly one point. 5

Or

A metric space (X, d) is separable if and only if it is second countable.

5. (a) Define a homeomorphism. 1

(b) Let (X, d) and (Y, ρ) be metric spaces, $f : X \to Y$ be a continuous function and $A \subseteq X$. Then show that the restriction f_A is continuous on A. 2

(c) Let (X, d) and (Y, ρ) be metric spaces. Then prove that a function $f : X \to Y$ is continuous if and only if $f^{-1}(F)$ is closed in X, whenever F is closed in Y. 5

(d) Show that a closed subset of a compact metric space is compact. 5

Or

Prove that a metric space is sequentially compact iff it has the Bolzano-Weierstrass property.

B : Statistics

(*Marks* : 40)

6. *(a)* Mention the defects of classical probability. 1

(b) Ten letters to each of which corresponds to one envelop are placed in the envelops at random. What is the probability that all letters are not placed in right envelops? 2

(c) Define independent events. Prove that if A and B are two independent events, then A and B' are also independent, where B' is the complement of B. 3

Or

Two urns contain 3 white, 7 red, 15 black and 10 white, 6 red, 9 black balls respectively. One ball is drawn from each urn. Find the probability that both the balls are of the same colour.

(d) The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40%, and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly? 4

7. (a) Define mean square deviation. 1

(b) (i) Write down the expression for variance of combined series. 1

(ii) The first of the two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15·6 and standard deviation $\sqrt{13 \cdot 44}$, find the standard deviation of the second group. 3

8. (a) Show that correlation coefficient is independent of change of origin and scale. 2

(b) Find the angle between two lines of regression. 2

(c) The variables X and Y are connected by the equation $aX + bY + c = 0$. Show that the correlation between them is -1 if the signs of a and b are alike and $+1$ if they are different. 3

Or

Find the equation of two lines of regression for the following data :

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

9. *(a)* What is the relation between binomial and Poisson distribution? Describe about the Poisson process. 1+2=3

(b) Six coins are tossed 6400 times. Using Poisson distribution, find the approximate probability of getting six heads r times. 2

(c) If X and Y are independent Poisson variates such that $P(X=1)=P(X=2)$ and $P(Y=2)=P(Y=3)$, then evaluate the variance of $X-2Y$. 3

(d) Find the probability density function of the normal distribution with mean 0 and unit variance. 4

Or

Discuss about the chief characteristics of the normal distribution and normal probability curve.

10. *(a)* Write down the different mathematical models for time series. 2

(b) Explain the method of curve fitting by the principle of least squares. 4

Or

Find the linear trend equation by the method of least squares from the following table. The table is with figures of production (in thousand tons) of a sugar factory :

Year	1999	2000	2001	2002	2003	2004	2005
Production	77	88	94	85	91	98	90

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