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6 SEM TDC MTH M 3

2019

(May)

MATHEMATICS

(Major)

Course : 603

[(A) Algebra—II and (B) Partial Differential Equations]

Full Marks : 80
Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Algebra—II

(*Marks* : 40)

1. *(a)* Write when an automorphism is called an outer automorphism. 1

(b) Define automorphism of a group G. 2

(c) Show that if G be a non-Abelian group, then the map $f: G \to G$ such that $f(x) = x^{-1}$ is not an automorphism. 4

Or

Let $f: G \to G$ be a homomorphism and f commutes with every inner automorphism of G. Show that G/K is Abelian, where $K = \{x \in G : f^2(x) = f(x)\}$ is a normal subgroup of G.

(d) Let G be an infinite cyclic group. Determine Aut$G$. 6

Or

Let H_1, H_2 be normal subgroups of G. Then show that G is an external direct product of H_1 and H_2 if and only if $G = H_1 H_2$ and $H_1 \cap H_2 = \{e\}$.

2. (a) Define unit element in a ring. 1

(b) Define a null ring. 1

(c) Write when a ring is called a ring with zero divisors. 1

(d) Prove that a non-zero finite integral domain is a field. 5

Or

Prove that a commutative ring R is an integral domain if and only if for all $a, b, c \in R$, $a \neq 0$, $ab = ac \Rightarrow b = c$.

(e) Prove that if in a ring R with unity, $(xy)^2 = x^2y^2$ for all $x, y \in R$, then R is commutative. 5

Or

Prove that if A and B are two ideals of R, then $A + B$ is an ideal of R, containing both A and B.

3. (a) Define kernel of a ring homomorphism. 1

(b) A field has only two ideals. Write them. 1

(c) Prove that if $f: R \to R'$ be an onto homomorphism, then R' is isomorphic to a quotient ring of R. 6

Or

Show that the relation of isomorphism in rings is an equivalence relation.

(d) Prove that any ring can be imbedded into a ring with unity. 6

Or

Let R be a commutative ring. Prove that an ideal P of R is prime if and only if $\frac{R}{P}$ is an integral domain.

(B) Partial Differential Equations

(*Marks* : 40)

4. (a) Identify the non-homogeneous equation(s) from the following : 1

(i) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2u$

(iii) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

(iv) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y} = 0$

(b) Define singular integral of a partial differential equation. 1

(c) Write the solution of the partial differential equation of the form $f(p, q) = 0$. 1

(d) Form the partial differential equation from $x + y + z = f(x^2 + y^2 + z^2)$ by eliminating arbitrary function f. 2

(e) Solve (any *three*) : 5×3=15

(i) $yzp + zxq = xy$

(ii) $(x + y)(p - q) = z$

(iii) $z(x+y)p + z(x-y)q = x^2 + y^2$

(iv) $(x+2z)p + (4zx - y)q = 2x^2 + y$

(v) $y^2(x-y)p + x^2(y-x)q = z(x^2 + y^2)$

(vi) $p - qy\log y = z\log y$

5. (a) Write the condition when two first-order partial differential equations are compatible. 1

(b) By applying Charpit's method, solution of a partial differential equation of any degree can be found. State true or false. 1

(c) Solve (any *three*) : 6×3=18

(i) $p = (z + qy)^2$

(ii) $z^2 = pqxy$

(iii) $(p^2 + q^2)y = qz$

(iv) $pq = xz$

(v) $z = px + qy + p^2 + q^2$

(vi) $p_3 x_3 (p_1 + p_2) + x_1 + x_2 = 0$

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