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6 SEM TDC MTH M 2

2018

(May)

MATHEMATICS

(Major)

Course : 602

(Discrete Mathematics and Graph Theory)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : DISCRETE MATHEMATICS

(*Marks* : 45)

1. Answer the following questions : 1×5=5

(a) $a_n = A \cdot 4^n + B \cdot 3^n$ is the solution of the recurrence relation $a_n - 7a_{n-1} + 12a_{n-2} = 0$. Write true or false.

(*b*) In a lattice, if $a \leq b$ and $c \leq d$, then $a \vee c \leq b \vee d$. Is it true?

(*c*) If

$$L_1 = \{2, 3, 4, 9, 36\},$$
$$L_2 = \{1, 2, 3, 4, 9, 36\}$$

and $L = \{(1, 2, 3, 4, 6, 9, 36), I\}$; then find whether L_1 and L_2 are sub-lattices of L or not.

(*d*) Give an example of an infinite lattice without 0 and 1.

(*e*) For a Boolean algebra B such that $a, b, c \in B$. Then show that

$$a \leq b \Rightarrow a + b \cdot c = b \cdot (a + c)$$

2. Answer the following questions : 2×3=6

(*a*) Let $(L, \vee, \wedge)$ be a distributive lattice. Then show that for any $a, b, c \in L$, $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$.

(*b*) Show that the elements 0 and 1 are unique in a Boolean algebra B.

(*c*) Find all sublattices of (D_{12}, I). Find one subset of D_{12} which is not a sublattice of it.

3. Answer any *two* of the following questions : 3×2=6

(a) Show that every chain $(L, \leq)$ is a distributive lattice.

(b) Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$; $a_0 = 3$, $a_1 = 0$.

(c) For all $a, b \in B$; B being a Boolean algebra, show that

(i) $(a+b)' = a' \cdot b'$

(ii) $(a \cdot b)' = a' + b'$

4. Answer any *two* of the following quetions : 5×2=10

(a) If m is a positive integer divisible by the square of a prime, show that D_m is not a Boolean algebra.

(b) Define minterm and maxterm with example(s). Obtain the sum-of-product canonical form for the Boolean expression

$$\overline{[\overline{(x_1 x_2)}\, x_3]}\ \overline{[(\bar{x}_1 + x_3)(\bar{x}_2 + \bar{x}_3)]}$$

(c) Find a minimal sum-of-products representation of the following Boolean function using Karnaugh map :

$$f(a,\ b,\ cd) = abcd + ab\bar{c}\bar{d} + ab\bar{c}d + a\bar{b}\bar{c}\bar{d} + \bar{a}bcd$$

(d) Define a bridge circuit. Draw a bridge circuit for the following function :

$$f = (x'u + x'v's + yu + yv's)(x' + z + w' + v's)(y + z + w' + u)$$

5. Answer any *three* of the following questions : 6×3=18

(a) Solve the recurrence relation

$$a_n - 4a_{n-1} + 3a_{n-2} = 3n^2 - 3n + 1$$

(b) If $G(x)$ is the generating function for $a_0, a_1, a_2, \ldots$, then find a generating function for $xG(x)$. If A implies a 2×2 matrix $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, then evaluate A^n using recurrence relation.

(c) Define Boolean function. Express the Boolean function

$$f(x, y, z) = (x + y) \cdot (x + z) + y + z'$$

in its disjunctive normal form.

(d) Show that (D_m, I) is a Boolean algebra, where m is the product of distinct primes.

(e) Write down the importance of special function. A logic circuit has $n = 4$ input devices A, B, C and D. Find the special sequences for A, B, C and D with their complements.

Or

Convert $f(x_1, x_2, x_3) = \pi(0, 2, 4, 5)$ into its canonical product-of-sums form.

B : GRAPH THEORY

(*Marks* : 35)

6. Answer the following questions : $1\times3=3$

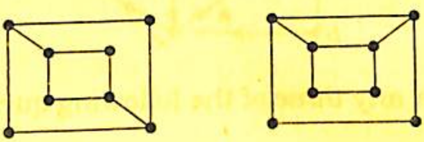
(a) Define a complex graph.

(b) State the Handshaking theorem.

(c) What do you mean by nullity of a graph G having n vertices, m edges and k components?

7. Answer the following questions : $2\times2=4$

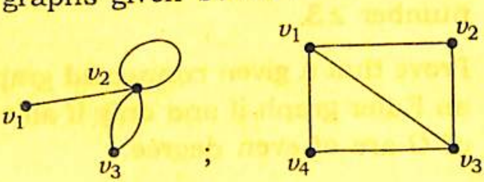
(a) Explain, why the following two graphs are not isomorphic :



(b) Describe briefly the Konigsberg's Bridge problem and produce a graph of it.

8. Answer any *two* of the following questions : $5\times2=10$

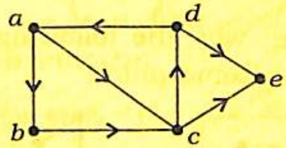
(a) Use adjacency matrix to represent the graphs given below :



(b) Draw the diagraph G corresponding to the adjacency matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

(c) Write down the practical uses of adjacency matrix and write the adjacency structure for the following graph :



9. Answer any *three* of the following questions : 6×3=18

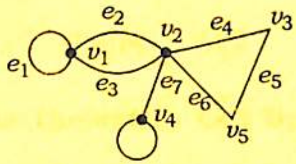
(a) Let G be a graph of order $p \geq 3$. If $\deg v \geq p/2$ for every vertex v of G, then show that G is Hamiltonian.

(b) Prove that in a complete graph with n vertices there are $(n-1)/2$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3.

(c) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree.

(d) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2.

(e) Define incidence matrix and find the incidence matrix to the graph



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