Total No. of Printed Pages—4

6 SEM TDC MTH M 3

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(May)

MATHEMATICS

(Major)

Course : 603

[(A) Algebra—II and (B) Partial Differential Equations]

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Algebra—II

(*Marks* : 40)

1. (a) Write when an isomorphic mapping of a group becomes an automorphism. 1

(b) Define inner automorphism of a group. 2

(c) Show that $f : G \to G$ such that $f(x) = x^{-1}$ is an automorphism if and only if G is Abelian. 5

Or

Let T be an automorphism of G. Show that $O(Ta) = O(a)$, $\forall\, a \in G$; and deduce that $O(bab^{-1}) = O(a)$, $\forall\, a, b \in G$.

(d) Show that set $I(G)$ of all inner automorphisms of G is a subgroup of aut (G). 5

Or

Let a group G is an internal direct product of its subgroups H and K. Show that H and K have only the identity in common.

2. (a) Write an example of a commutative ring with unity. 1

(b) Define a field. 1

(c) Show that a field has no proper ideals. 3

(d) Show that a field is an integral domain. 4

Or

Show that the intersection of two subrings is a subring.

(e) Prove that the set of integers is an integral domain with respect to addition and multiplication. 4

3. (a) Define prime ideal. 1

(b) If R / S is a ring of residue classes of S in R, show that R / S is commutative if R is commutative. 4

(c) Let $f: R \to R'$ be an onto homomorphism, where R is a ring with unity. Show that $f(1)$ is unity of R'. 4

(d) Let R be a commutative ring. Show that an ideal P of R is prime if and only if R/P is an integral domain. 5

Or

Let R be a commutative ring with unity. Show that an ideal M of R is maximal ideal of R if and only if R/M is a field.

(B) Partial Differential Equations

(*Marks* : 40)

4. (a) Write the order of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \left(\frac{\partial z}{\partial y}\right)^4 = 0$$ 1

(b) Write Lagrange's auxiliary equations for the equation $2(p+q) = z$. 1

(c) Solve : 2

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2 y^2 z^2}$$

(d) Solve (any *two*) : 3×2=6

(i) $xp + yq = z$

(ii) $y^2 p + x^2 q = x^2 y^2 z^2$

(iii) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

(*Turn Over*)

(e) Solve/Answer (any *two*) : 5×2=10

(i) $(1+y)p+(1+x)q=z$

(ii) $(y+z)p+(z+x)q=x+y$

(iii) Find the equation of the surface satisfying $4yzp+q+2y=0$ and passing through $y^2+z^2+1=0$, $x+z=2$.

5. *(a)* Let $f(x_i)=0$ be a partial differential equation having n independent variables. Then write the number of constants that appear in the solution. 1

(b) Define particular integral of $f(x, y, z, p, q)=0$. 1

(c) Write when two first-order partial differential equations are compatible. 2

(d) Show that the equations $xp=yq$ and $z(xp+yq)=2xy$ are compatible. 6

Or

Find a complete integral of $px+qy=pq$.

(e) Find the complete integral of $P_1^3+P_2^2+P_3=1$ using Jacobi's method. 5

(f) Find the complete integral of (any *one*) : 5

(i) $(p^2+q^2)x=pz$

(ii) $px+qy+pq=0$

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