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(May)

MATHEMATICS

(Major)

Course : 603

[(A) Algebra—II and (B) Partial Differential Equations]

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Algebra—II

(*Marks* : 40)

1. *(a)* Define a trivial automorphism. 1

(b) Write the condition when a group G has a non-trivial automorphism. 1

(c) Show that if G is a non-Abelian group, then $f : G \to G$, such that $f(x) = x^{-1}$ is not an automorphism. 3

(d) If $f : G \to G$ such that $f(a) = a^n$ is an automorphism, then show that

$$a^{n-1} \in Z(G), \forall a \in G$$ 3

(e) If G be an infinite cyclic group, then determine Aut G. 5

Or

Let H_1, H_2 be normal in G. Then prove that G is an internal direct product of H_1 and H_2 if and only if—

(i) $G = H_1 H_2$;

(ii) $H_1 \cap H_2 = \{e\}$.

2. (a) Write when a ring is called a ring with unity. 1

(b) Give an example of a ring which is not an integral domain. 1

(c) State True or False : 1

The product AB of any two ideals A and B of a ring R is not an ideal of R.

(d) Prove that a commutative ring R is an integral domain if and only if

$$a, b, c \in R \, (a \neq 0), \; ab = bc \Rightarrow b = c$$ 5

Or

Prove that a finite integral domain is a field.

(e) Prove that a non-empty subset s of a ring R is a subring of R if and only if

$$a, b \in s \Rightarrow ab,\ a-b \in s$$ 5

Or

If A and B are two ideals of R, then prove that $A+B$ is an ideal of R containing both A and B.

3. *(a)* Write the maximal ideal of a field F. 1

(b) Define quotient ring. 2

(c) Prove that if $f: R \to R'$ be an onto homomorphism, then R' is isomorphic to a quotient ring. 6

Or

Prove that any ring can be imbedded into a ring with unity.

(d) Let R be a commutative ring. Prove that an ideal P of R is a prime ideal if for two ideals A, B of R, $AB \subseteq P \Rightarrow$ either $A \subseteq P$ or $B \subseteq P$. 5

(B) Partial Differential Equations

(*Marks* : 40)

4. (*a*) Write the degree of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial z}{\partial y}=0$$ 1

(*b*) Write the Lagrange's auxiliary equations for the equation

$$y^2 p - xyq = x(z-2y)$$ 1

(*c*) Form the partial differential equation by eliminating a and b from

$$z = a(x+y)+b$$ 2

(*d*) Solve any *two* of the following : 3×2=6

(*i*) $a(p+q)=z$

(*ii*) $zp=-x$

(*iii*) $yp+xq=z-1$

(*e*) Solve any *two* of the following : 5×2=10

(*i*) $(1+y)p+(1+x)q=z$

(*ii*) $xzp+yzq=xy$

(*iii*) $xp+zq+y=0$

(*iv*) $xp-yq=xy$

5. (a) Define complete integral of

$$f(x, y, z, p, q) = 0$$ 1

(b) Write the complete solution of

$$z = px + qy + \log(pq)$$ 1

(c) Write the Charpit's auxiliary equations for the equation $3p^2 = q$. 2

(d) Show that $p^2 + q^2 = 1$ and

$$(p^2 + q^2)x = pz$$

are compatible. 6

Or

Find the complete integral of

$$p_1 + p_2 + p_3 - p_1 p_2 p_3 = 0$$

by Jacobi's method, where

$$p_1 = \frac{\partial z}{\partial x_1},\ p_2 = \frac{\partial z}{\partial x_2},\ p_3 = \frac{\partial z}{\partial x_3}$$

(e) Solve any *two* of the following : 5×2=10

(i) $zpq = p + q$

(ii) $p^2 - y^2 q = y^2 - x^2$

(iii) $q = (z + px)^2$

(iv) $pxy + pq + qy = yz$

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