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6 SEM TDC MTH M 4 (A/B)

2017

(May)

MATHEMATICS

(Major)

Course : 604

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

[**(a) Financial Mathematics**

(b) Operations Research]

(a) Financial Mathematics

(*Marks* : 45)

1. (a) Define demand function and supply function. 2

(b) The supply and demand functions for a commodity are

$$q^S(p) = 12p - 4, \; q^D(p) = 8 - 4p$$

If an excise tax of T is imposed, then what are the selling price and quantity sold, in equilibrium? 3

2. Describe the Cobweb model. 5

Or

Consider a market in which the supply and demand sets are

$$S = \{(q, p) \mid q = 3p - 7\}$$
$$D = \{(q, p) \mid q = 38 - 12p\}$$

Write down the recurrence equations which determine the sequence p_t of price, assuming that the suppliers operate according to the Cobweb model. Find the explicit solution given that $p_0 = 4$ and describe in words how the sequence p_t behaves. Write down a formula for q_t, the quantity on the market in the year t. 5

3. (a) Suppose the cost function is $C(q) = 9 + 5q$ and the price function is $P(q) = 6 - 0{\cdot}01q$. Then write the profit function $\pi(q)$. 1

(b) The supply set S consists of pairs (q, p) such that $2q - 5p = 14$ and a demand set D consists of pairs (q, p) such that $3q + p = 72$. An excise tax T per unit is imposed. Determine when the revenue will be maximum. 4

4. (a) Define elasticity of demand. 3

(b) State the difference between competition and monopoly. 3

(c) Consider an efficient small firm with the cost function

$$C(q) = q^3 - 10q^2 + 100q + 196$$

that can produce maximum of 10 units per week. Determine their—

(i) fixed cost;

(ii) profit function;

(iii) startup point;

(iv) breakeven point;

(v) supply set. 4

5. (a) Define saddle point. 1

(b) If $f(x, y) = x^3 - y^3 - 2xy + 1$, then find and classify the critical points of f. 5

(c) Find the maximum value of the function

$$f(x, y) = 6 + 4x - 3x^2 + 4y + 2xy - 3y^2$$ 4

6. (a) Define a technology matrix. 2

(b) The supply function for a good is

$$q^S(p) = ap^3 + bp^2 + c$$

for some constants a, b, c. When $p = 1$, the quantity supplied is 1, when $p = 2$, the quantity supplied is 11, when $p = 3$, the quantity supplied is 35. Find the constants a, b, c. 4

(c) The matrix of return for an investor is

$$R = \begin{pmatrix} 1\cdot05 & 0\cdot95 \\ 1\cdot05 & 1\cdot05 \\ 1\cdot37 & 1\cdot42 \end{pmatrix}$$

Show that the portfolio $Y = (500 \;\; 10000 \;\; 1000)$ is riskless. What return is the investor guaranteed? 4

(b) Operations Research

(*Marks* : 35)

7. (a) State True or False : 1

Operations research practioners can predict about the future events.

(b) What is OR? Write a short note on application of OR. 1+3=4

Or

Write a short note on the limitations of operations research. 4

8. (a) Define assignment problem. 1

(b) Explain the difference between a transportation problem and an assignment problem. 2

(c) Consider the problem of assigning five operators to five machines. The assignment costs are given below :

Operators

Machines		I	II	III	IV	V
	A	10	5	13	15	16
	B	3	9	18	3	6
	C	10	7	2	2	2
	D	5	11	9	7	12
	E	7	9	10	4	12

Assign the operators to different machines so that total cost is minimized. 7

9. (a) Explain the concept of dynamic programming and the relation between dynamic and linear programming approaches. 3

(b) Use dynamic programming to solve the following linear programming problem : 7

Maximize $Z = 3x_1 + 5x_2$

subject to

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$3x_1 + 2x_2 \leq 18$$
$$\text{and } x_1, x_2 \geq 0$$

Or

Solve the following LPP by the method of dynamic programming : 7

Maximize $Z = 2x_1 + 5x_2$

subject to

$$2x_1 + x_2 \le 430$$
$$2x_2 \le 460$$
and $x_1, x_2 \ge 0$

10. (a) Fill in the blank : 1

_____ programming is an extension of the linear programming in which feasible solution must have integer value.

(b) Explain the basic difference between a pure and mixed integer programming problems. 2

(c) Solve the following all integer programming problem using Gomory's cutting plane method : 7

Maximize $Z = x_1 + 2x_2$

subject to

$$2x_2 \le 7$$
$$x_1 + x_2 \le 7$$
$$2x_1 \le 11$$

and $x_1, x_2 \ge 0$ and integers.

(*Continued*)

Or

Use Gomory's cutting plane method to solve the following problem : 7

Maximize $Z = x_1 - x_2$

subject to

$$x_1 + 2x_2 \leq 4$$
$$6x_1 + 2x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

and are integers.

GROUP—B

[**(a) Space Dynamics**
(b) Relativity]

(a) Space Dynamics

(*Marks* : 40)

1. *(a)* Define spherical angle. 1

(b) Fill in the blank : 1

Number of great circle through two given points is _____, if the two points are not the extremities of a diameter.

(c) Show that the sum of the three angles of a spherical triangle is greater than two right angles but less than six right angles. 2

(*Turn Over*)

(d) Prove the sine-cosine formula : 4

$$\sin b \cos C = \sin a \cos c - \cos a \sin c \cos B$$

(e) In a spherical triangle *ABC*, if θ, ϕ, ψ be the arcs bisecting the angles *A*, *B*, *C* respectively and terminated by opposite sides, show that

$$\cot\theta \cos\frac{A}{2} + \cot\phi \cos\frac{B}{2} + \cot\psi \cos\frac{C}{2} = \cot a + \cot b + \cot c$$ 5

Or

In a spherical triangle *ABC*, prove that

$$\frac{\sin^2 a + \sin^2 b + \sin^2 c}{\sin^2 A + \sin^2 B + \sin^2 C} = \frac{1 - \cos a \cos b \cos c}{1 + \cos A \cos B \cos C}$$ 5

2. (a) What is astronomical latitude? 1

(b) Define celestial equator and observer's meridian. 1+1=2

(c) What is the RA of the Sun when it is on the summer solstice? 1

(d) Write short notes on any *two* of the following : 2×2=4

(i) Hour angle

(ii) Equinoxes

(iii) Elements of the orbit in space

(e) Discuss the ecliptical coordinate system. 4

Or

If (α, δ) and (λ, β) are respectively the equatorial and ecliptic coordinates of a star, then prove that

$$\sin\beta = \cos\varepsilon \sin\delta - \sin\varepsilon \cos\delta \sin\alpha$$

$$\text{and } \tan\lambda = \frac{\sin\varepsilon \tan\delta + \cos\varepsilon \sin\alpha}{\cos\alpha}$$

where ε is the obliquity of the ecliptic. 4

(f) If H be the hour angle of a star of declination δ when its azimuth is A and H' when azimuth is $180° + A$, then prove that

$$\tan\phi = \tan\delta \frac{\cos\left(\frac{H' + H}{2}\right)}{\cos\left(\frac{H' - H}{2}\right)}$$

where ϕ is the latitude of the star. 5

3. *(a)* Define mean anomaly. 1

(b) In one-body problem, deduce the equation $r = \frac{a(1 - e^2)}{1 + e\cos\omega}$, where a is semimajor axis, e is the eccentricity, ω is the true anomaly of the moving particle at any position (r, θ). 5

Or

Derive an expression for the position of a body in an elliptic orbit. 5

(c) Establish the relation

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

where ν is true anomaly and E is eccentric anomaly. 4

Or

Deduce the Kepler's equation

$$M = E - e\sin E = n(t-\tau)$$ 4

(b) Relativity

(*Marks* : 40)

4. (a) State True or False : 1

It is possible to send out signals with a velocity greater than the velocity of light.

(b) Choose the correct answer : 1

Frame S' is moving with velocity v along x-axis relative to a stationary frame S with length l along x-axis. The length as observed in frame S' is

(i) $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ (ii) $l\sqrt{1-\frac{v^2}{c^2}}$

(iii) l (iv) $\frac{l}{v}$

(c) State two postulates of special theory of relativity. 2

(*d*) Write short note on any *one* of the following : 4

(*i*) Clock paradox

(*ii*) Length contraction

5. Show that the inverse of a Lorentz transformation is also a Lorentz transformation. 6

Or

If u and v are two velocities in the same direction and V their resultant velocity given by

$$\tanh^{-1}\frac{V}{c} = \tanh^{-1}\frac{u}{c} + \tanh^{-1}\frac{v}{c}$$

then deduce the law of composition of velocities from this equation. 6

6. Answer any *two* of the following : 3×2=6

(*a*) A particle with a mean proper life 1 μ sec moves through the laboratory at $2 \cdot 7 \times 10^{10}$ cm/sec. What will be its life as measured by an observer in the laboratory?

(*b*) A rod of length 1 m, when the rod is in a satellite moving with velocity $0 \cdot 8c$ relative to laboratory, what is the length of the rod as determined by an observer (*i*) in the satellite and (*ii*) in the laboratory?

(*c*) Why is the velocity of light called fundamental velocity?

7\. (a) Choose the correct answer : 1

The relation between momentum and energy is

(i) $E^2 = p^2c^2 + m_0^2c^2$

(ii) $E^2 = p^2c^2 - m_0^2c^4$

(iii) $E^2 = p^2c^2 + m_0^2c^4$

(iv) $E^2 = p^2c^2 - m_0^2c^2$

(b) What is space-like interval? 1

(c) Show that the rest mass of a particle of momentum P and kinetic energy K is

$$m_0 = \frac{P^2c^2 - K^2}{2Kc^2}$$ 3

Or

Calculate the velocity at which the mass of a particle becomes 8 times its rest mass. 3

(d) How much electric energy could theoretically be obtained by annihilation of 1 g of matter? 3

8\. Answer any *two* of the following : 6×2=12

(a) Establish the Einstein mass-energy relation $E = mc^2$.

(b) Find the transformation laws of density in relativistic mechanics.

(c) Calculate the rest mass of a particle whose momentum is 130/c MeV, when its kinetic energy is 50 MeV.

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