6 SEM TDC MTH M 1

2014

(May)

MATHEMATICS

(Major)

Course : 601

(A : Metric Spaces and B : Statistics)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Metric Spaces

(*Marks* : 40)

1. *(a)* Define diameter of a set. 1

(b) Prove that in a metric space (X, d), the empty set ϕ and the whole space X are closed. 2

(c) Prove that a set A is open $\Leftrightarrow A = \text{int}, (A)$. 3

(*d*) Let X be any non-empty set and d a function defined on X such that $d: X \times X \to R$ defined by

$$d(x, y) = 0, \text{ if } x = y$$
$$= 1, \text{ if } x \neq y$$

Then prove that d is a metric on X. 4

(*e*) Define open set. Prove that for a given metric space (X, d), a subset G of X is open $\Leftrightarrow G$ is a union of open spheres. 1+4=5

Or

Let X be a metric space, and let A be a subset of X. If x is a limit point of A, show that each open sphere centred on x contains an infinite number of distinct points of A. Use this result to show that a finite subset of X is closed. 3+2=5

2. (*a*) Let X be a metric space and A a subset of X. Prove that A is dense $\Leftrightarrow$ the only closed superset of A is X. 3

(*b*) Let X be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, show that

$$d(x_n, y_n) \to d(x, y)$$ 4

(c) Given a complete metric space (X, d) and Y a subspace of X, prove that Y is complete $\Leftrightarrow Y$ is closed. 5

3. (a) Let X and Y be metric space and f a mapping of X into Y. Prove that f is continuous at x_0 iff

$$x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$$ 3

(b) Define uniform continuity. If (X, d_1) and (Y, d_2) are two metric spaces and f, a mapping of X into Y, i.e., $f : X \to Y$, then prove that f is continuous $\Leftrightarrow f^{-1}(G)$ is open in X whenever G is open in Y.

1+4=5

Or

Let X be a metric space, let Y be a complete metric space and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y, then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y. 5

4. Define a sequentially compact metric space. Prove that a metric space is sequentially compact $\Leftrightarrow$ it has the Bolzano-Weierstrass property. 1+4=5

B : Statistics

(*Marks* : 40)

5. (*a*) If A is a certain event, then what is its probability of happening? 1

(*b*) Define independent and mutually exclusive events. Can two events be mutually exclusive and independent simultaneously? 2

(*c*) A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B. 3

(*d*) Prove that if A, B and C are random events in a sample space and if A, B and C are pair-wise independent and A is independent of $(B \cup C)$, then A, B and C are mutually independent. 4

6. (*a*) Define quartile deviation. 1

(*b*) Find the standard deviation and coefficient of variation from the following distribution : 4

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	10	15	25	25	10	10	5

7. (a) When do two lines of regression reduce to one? 2

(b) Calculate Karl Pearson's coefficient of correlation and the regression equations from the following data : 5

Age of husband	18	19	20	21	22	23	24	25	26	27
Age of wife	17	17	18	18	18	19	19	20	21	21

8. (a) Write the condition under which a binomial distribution becomes a symmetrical distribution. 1

(b) If X is a random variate such that $P(X=1)=P(X=2)$, find $P(X=4)$. 2

(c) Prove that for Poisson distribution

mean = variance 4

(d) Mention the conditions under which binomial distribution tends to normal distribution. Prove that for the normal distribution the mean deviation from the mean $\mu = \frac{4}{5}\sigma$, where σ is standard deviation. 2+3=5

Or

Find the mean and variance of the normal distribution by differentiating w.r.t. μ, twice the identity

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$$

Find also the variance by differentiating with respect to σ alone. 5

9. Define time-series. Mention its important components and describe a method of smoothing of time-series. 1+2+3=6

Or

What is meant by trend of a time-series? Explain how the 'principle of least squares' is used to estimate trend in a time-series. 2+4=6

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