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6 SEM TDC MTH M 2

2014

(May)

MATHEMATICS

(Major)

Course : 602

(Discrete Mathematics and Graph Theory)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) DISCRETE MATHEMATICS

(*Marks* : 45)

1. Answer the following questions : 1×5=5

(a) What is the general solution of a recurrence relation if 4, 4 are the roots of the corresponding characteristic equation?

(b) Lattice is called as an algebraic system. Why?

(c) Give an example of a poset which has no maximal element.

(d) Under what condition a lattice is said to be complete?

(e) Find the dual of the Boolean function

$$f = x(y'z' + yz)$$

2. Answer the following questions : $2\times3=6$

(a) What do you mean by initial condition for a recurrence relation? Solve the recurrence relation

$$a_n - da_{n-1} = 0, \ a_0 = 4$$

(b) Let $(L, \leq)$ be a lattice. Then show that

$$a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b \ ; \ a, b \in L$$

(c) Let $L_1 = \{2, 3, 4, 9, 36\}$, $L_2 = \{1, 2, 3, 4, 9, 36\}$ and $L_3 = \{1, 2, 3, 6\}$. Examine whether L_1, L_2, L_3 are sublattices of

$$L = \{(1, 2, 3, 4, 6, 9, 36), \backslash\}; \ `\backslash'$$

being the divisibility relation.

3. Answer any *two* of the following questions : $3\times2=6$

(a) Show that the complement of an element a in a Boolean algebra B is unique.

(b) Draw switching circuits for the function

$$f = a(bc + d(e + f))$$

(c) Express Boolean function $f = x + y'z$ in a sum of minterms.

4. Answer any *two* of the following questions : 5×2=10

(a) Define generating function. If

$$A_{2\times 2} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$$

evaluate A^n using recurrence relation. Also find A^{30}.

(b) For $a, b \in Z^+$, the set of positive integers, show that $\text{lcm}(a, \text{gcd}(a, b)) = a$ and $\text{gcd}(a, \text{lcm}(a, b) = a$.

(c) Define lattice isomorphism. Let L_1 be the lattice D_6 (divisor of 6) and L_2 be the lattice $(P(s), \subseteq)$ such that $s = \{a, b\}$. Show that L_1 and L_2 are isomorphic.

5. Answer any *three* of the following questions : 6×3=18

(a) Find the total solution of the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + 3n$; $a_0 = 1$, $a_1 = 6$.

(b) Show that $(D_m, 1)$ is a distributive lattice.

(c) If $X = \{a, b, c\}$ and $f : h(X) \to B_2$ (B_2 is a Boolean algebra with 2 elements) is defined by

$$f(a) = \begin{cases} 0 & ; \quad a \notin A \\ 1 & ; \quad a \in A \end{cases}$$

show that f is homomorphism from $h(X)$ to B_2.

(d) A logic circuit is represented by the Boolean function

$$f = (a + b' + d)(a + b + c')(a + c + d)$$

Simplify the circuit and represent it.

(B) GRAPH THEORY

(*Marks* : 35)

6. Answer the following questions : 1×3=3

(a) What do you mean by a simple graph?

(b) Does there exist a 4-regular graph on 5 vertices?

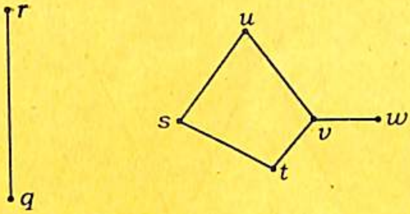
(c) Define adjacency matrix with example.

7. Answer the following questions : 2×2=4

(a) Show that the maximum number of edges in a simple graph with n vertices is

$$\frac{1}{2}n(n-1)$$

(b) Find the vertex sets of components and cut vertices of the following graphs :



8. Answer any *two* of the following questions : $5 \times 2 = 10$

(a) Find the number of edges in a complete bipartite graph of n vertices.

(b) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

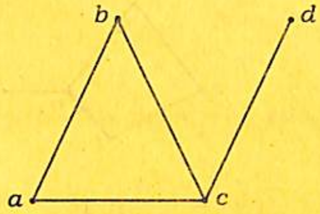
(c) Show that a graph which is Hamiltonian may not be Eulerian and vice-versa.

9. Answer any *three* of the following questions : $6 \times 3 = 18$

(a) Show that a simple graph G with n vertices $(n \geq 2)$ has a Hamiltonian circuit if $d(u) + d(v) \geq n$ for all non-adjacent vertices u, v in G.

(b) If the intersection of two paths in a graph G is a disconnected graph, show that the union of the two paths has at least one circuit.

(c) Define linked representation of a graph. Find a linked representation for the graph



(d) Show that a simple connected graph with n vertices and m edges is Hamiltonian if

$$m \geq \frac{1}{2}(n-1)(n-2)+2$$

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