6 SEM TDC MTH M 3

2014

(May)

MATHEMATICS

(Major)

Course : 603

(Algebra II and Partial Differential Equation)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

A : Algebra II

(*Marks* : 40)

1. (a) Let G be a group and the identity map $I : G \to G$, such that $I(x) = x$. Write the name of this automorphism. 1

(b) Define inner automorphism. 3

(c) Let $f : G \to G$ such that $f(x) = x^n$ is an automorphism, where n is some fixed integer. Show that $a^{n-1} \in Z(G)$, for all $a \in G$. 3

(d) Prove that the set $I(G)$ of all inner automorphisms of G is a subgroup of Aut G, where Aut G is the set of all automorphisms of G. 6

Or

Let G be an infinite cyclic group. Determine Aut G, where Aut G is the set of all automorphisms of G.

2. (a) Set of all even integers forms a commutative ring under addition and multiplication without unity. Write true or false. 1

(b) Write the condition when a ring R is a commutative ring. 1

(c) In a ring R, show that

$$a(b-c) = ab - ac, \ \forall \ a, b, c \in R$$ 2

(d) Prove that a field is an integral domain. 4

Or

If A is an ideal of a ring R with unity such that $1 \in A$, then show that $A = R$.

(e) Prove that, if A and B are two ideals of R, then $A + B$ is an ideal of R, containing both A and B. 5

Or

Prove that the product AB of any two ideals A and B of a ring R is an ideal of R.

3. *(a)* If R is a ring, I is an ideal and $I = R$, then to which type of ring R/I is isomorphic? 1

(b) Let $f: R \to R'$. Then define kernel of f. 2

(c) Define maximal ideal of a ring. 2

(d) If $f: R \to R'$ is a homomorphism, then show that $f(0) = 0'$ and $f(-a) = -f(a)$. 4

Or

Let R be a commutative ring with unity. Show that every maximal ideal of R is prime.

(e) Let A, B be two ideals of a ring R, then show that

$$\frac{A+B}{A} \cong \frac{B}{A \cap B}$$ 5

Or

Let R be a commutative ring. An ideal P of R is prime if and only if $\frac{R}{P}$ is an integral domain. Prove it.

B : Partial Differential Equation

(*Marks* : 40)

4. (*a*) Write the order of the partial differential equation

$$y\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = z$$ 1

(*b*) Form the partial differential equation by eliminating arbitrary constants m and n from the relation

$$z = m(x+y) + n$$ 2

(*c*) Show that the curves represented by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

lie completely on the surface $z = \phi(x, y)$ whose differential is $Pp + Qq = R$. 3

(*d*) Solve any *one* of the following equations by Lagrange's method : 4

(*i*) $y^2 p - xyq = x(z - 2y)$

(*ii*) $yzp + 2xq = xy$

(*e*) Solve any *one* of the following : 5

(*i*) $xp - yq = xy$

(*ii*) $(y - zx)p + (x + yz)q = x^2 + y^2$

(f) Find the equation of surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1$ and $x + z = 2$. 5

Or

Solve $p + q = x + y + z$

5. *(a)* Charpit's equations give $p = a$ and $q = b$ for the equation $z = px + qy + p^2 + q^2$. Write the complete integral. 1

(b) Write the general form of the complete integral of the equation $f(p, q) = 0$. 1

(c) Write the Charpit's auxiliary equations for the equation $px + qy = pq$. 2

(d) For the arbitrary constants a and b, $z = ax + by + ab$ is the complete integral of the equation $z = px + qy + pq$. Obtain the singular integral of the equation. 2

(e) Find complete integral of any *two* equations of the following : 5×2=10

(i) $q = (z + px)^2$

(ii) $yzp^2 = q$

(iii) $q = px + p^2$

(iv) $pxy + pq + qy = yz$

(f) Solve any *one* of the following equations by Jacobi's method : 4

(i) $p^2 + q^2 = k^2$

(ii) $xz = pq$

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