6 SEM TDC MTH M 4

2014

(May)

MATHEMATICS

(Major)

Course : 604

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

[**(a) Financial Mathematics**

(b) Operations Research]

(a) Financial Mathematics

(*Marks* : 45)

1. *(a)* If $(q, p) = (g, z)$ is a point in the demand set D, what is the value of $q^D(p)$? 1

(b) Consider the demand and supply functions given by $a_1q + b_1p = c_1$ and $a_2q + b_2p = c_2$ respectively. When an

excise tax T is imposed, per unit of the item, establish that the selling price is not increased by a full amount T, but by a fraction of T. 4

2. Describe the Cobweb model. 5

Or

The supply and demand sets of the market of an item are

$$S = \{(q,\ p) : q = 8p - 35\}$$

and
$$D = \{(q,\ p) : q = 25 - 4p\}$$

respectively. The non-equilibrium price is ₹ 7. Find the recurrence equation and the quantity on the market in the year t. Discuss equilibrium.

3. *(a)* Define a stationary point of a function $y = f(x)$. 1

(b) The supply set S consists of pairs $(q,\ p)$ such that $2q - 5p = 14$ and a demand set D consists of pairs $(q,\ p)$ such that $3q + p = 72$. An excise tax T per unit is imposed. Determine when the revenue will be maximum. 4

4. (a) State True or False : 1

At the break-even point the marginal cost is roughly equal to the average cost.

(b) State the difference between competition and monopoly. 2

(c) A software firm can produce 100 softwares per year whose demand set is

$$D = \{(q,\ p) : q + 4p = 600\}$$

Find the maximum profit. 3

(d) Consider an efficient small firm with the cost function

$$C(q) = 900 + 80q - 13q^2 + 2q^3$$

that can produce maximum of 8 units per day. Determine their—

(i) fixed cost;

(ii) profit function;

(iii) startup point;

(iv) break-even point;

(v) supply set. 4

5. *(a)* Describe how prices are related to quantities. 3

(b) Show that

$$f(x, y) = 3x^2 + 2xy + 2y^2 - 160x - 120y + 18$$

has only one critical point and classify it. 3

(c) Find the critical points of the function

$$u(x, y) = y^3 + 3xy - x^3$$

and classify them. 4

6. *(a)* Describe the Leontief matrix. 2

(b) A factory makes two goods, grommets and widgets. To make ₹ 1 worth of grommets requires ₹ 0·1 worth of widgets, and to make ₹ 1 worth of widgets requires ₹ 0·15 worth of grommets and ₹ 0·05 worth of widgets. There is an external demand for ₹ 500 worth of grommets and ₹ 1,000 worth of widgets. What should be the total production of each commodity? 4

(c) The demand function for a commodity takes the form

$$q^D(p) = a + bp + \frac{c}{p}$$

where a, b, c are constants. When $p = 1$ the quantity demanded is 60; when $p = 2$ it is 40, and when $p = 4$ it is 15. Find the constants a, b and c. 4

(b) Operations Research

(*Marks* : 35)

7. (a) State one reason why Operations Research cannot be defined satisfactorily. 1

(b) Write a short note on limitations of Operations Research. 4

Or

Describe briefly the scope of Operations Research in financial management.

8. (a) Define assignment problem. 1

(b) Give a mathematical representation of an assignment model. 2

(c) Solve any *one* of the following : 7

(i) Solve the assignment problem :

	I	*II*	*III*	*IV*	*V*
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

(ii) Five wagons are available at stations *1*, *2*, *3*, *4* and *5*. These are required at five stations *I*, *II*, *III*, *IV* and *V*. The mileages between various stations are given by the table below. How should the wagons be transported so as to minimize the total mileage covered?

	I	*II*	*III*	*IV*	*V*
1	10	5	9	18	11
2	13	9	6	12	14
3	3	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

9. *(a)* What do you mean by dynamic programming? 1

(b) State two applications of dynamic programming. 2

(c) State Richard Bellman's principle of optimality. 2

(d) Solve the LPP by dynamic programming : 5

$$\text{Maximize} \quad Z = 2x_1 + 5x_2$$

subject to

$$2x_1 + x_2 \le 430$$
$$2x_2 \le 460$$
$$x_1, x_2 \ge 0$$

10. *(a)* What do you mean by Pure (all) Integer Programming Problem and Mixed Integer Programming Problem? 2

(b) State the properties of Gomory's All-Integer Algorithm. 2

(c) Describe the steps of Gomory's All-Integer Programming Algorithm. Give the flowchart. 6

Or

Solve the following Integer LP Problem using Gomory's cutting plane method :

Maximize $Z = x_1 + x_2$

subject to the constraints

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

and $x_1, x_2 \geq 0$ and are integers

GROUP—B

[(a) **Space Dynamics**

(b) **Relativity**]

(a) Space Dynamics

(*Marks* : 40)

1. Answer as directed : 1×2=2

(a) Choose the correct answer :

If A, B, C are the three angles of a spherical triangle, then

(i) $90^\circ < A + B + C < 360^\circ$

(ii) $180^\circ < A + B + C < 540^\circ$

(iii) $180^\circ < A + B + C < 360^\circ$

(b) Write down the statement of the four parts formula of a spherical triangle.

2. Prove that the sum of three sides of a spherical triangle is less than 2π. 3

Or

In spherical triangle ABC, if D is the middle point of AB, prove that

$$\cos a + \cos b = 2\cos\frac{c}{2}\cos CD$$

3. State and prove the sine formula of a spherical triangle. 4

Or

In any spherical triangle *ABC*, prove that

$$\sin a \cos B = \sin c \cos b - \cos c \sin b \cos A$$

4. In a spherical triangle *ABC*, if $a+b+c=180°$, then prove that

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1$$ 4

Or

Perpendiculars drawn from the angles *A*, *B*, *C* of a spherical triangle meet the opposite sides at *D*, *E*, *F*. Prove that

$$\tan BD \tan CE \tan AF = \tan DC \tan EA \tan FB$$

5. Define any *two* of the following : 1×2=2

(a) Vertical circles

(b) The ecliptic

(c) Circumpolar stars

6. Write short notes on any *two* of the following : 2×2=4

(a) Diurnal motion of heavenly bodies

(b) Cardinal points and equinoctial points

(c) Longitude and latitude of a star

7. If (α, δ) and (λ, β) are respectively the equatorial system and ecliptic system of coordinates of a star, then prove that

$$\sin\delta = \cos\varepsilon\sin\beta - \sin\varepsilon\cos\beta\sin\lambda$$

and $$\tan\alpha = \frac{\cos\varepsilon\sin\lambda - \sin\varepsilon\tan\beta}{\cos\lambda}$$

where ε is the obliquity of the ecliptic. 3+3=6

8. Two stars (α_1, δ_1) and (α_2, δ_2) have the same latitude, prove that

$$\sin(\alpha_1 - \alpha_2) = \tan\varepsilon(\cos\alpha_1 \tan\delta_2 - \cos\alpha_2 \tan\delta_1)$$ 5

Or

The hour angle of a star of declination δ is H_1 when it has the azimuth A. Again the hour angle is H_2 when it has the azimuth 180°+A. Prove that the latitude ϕ can be obtained from the equation

$$\tan\phi = \tan\delta \frac{\cos(H_2 + H_1)/2}{\cos(H_2 - H_1)/2}$$

9. Write down the statement of Newton's law of gravitation. 1

10. What are the six elements required for complete specification of a planetary orbit in space? 3

11. Derive an expression for velocity of a body in an elliptic orbit. 6

Or

Derive Kepler's 2nd and 3rd laws from Newton's law of gravitation.

(b) Relativity

(*Marks* : 40)

12. Choose the correct answer : $1 \times 2 = 2$

   (a) Frame S' is moving with velocity v along x-axis relative to a stationary frame S with length l along x-axis. The length as observed in frame S' is

   (i) $\dfrac{l}{\sqrt{1-\dfrac{v^2}{c^2}}}$

   (ii) $l\sqrt{1-\dfrac{v^2}{c^2}}$

   (iii) l

   (iv) $\dfrac{l}{v}$

(b) The relation between momentum and energy is

(i) $E^2 = p^2c^2 + m_0^2c^4$

(ii) $E^2 = p^2c^2 - m_0^2c^4$

(iii) $E^2 = p^2c^2 + m_0^2c^2$

(iv) $E^2 = p^2c^2 - m_0^2c^2$

13. Write True or False : 1×2=2

(a) The velocity of light c can be changed by adding to it a velocity smaller than c.

(b) Simultaneity is not absolute, but it is relative.

14. State the two postulates of special theory of relativity. 2

15. Write a short note on any *one* of the following : 2

(a) Time dilation

(b) Relative simultaneity

16. Find out the relativistic formulae for composition of velocities. Hence, show that the velocity of light is an absolute constant. 6

Or

Deduce the Lorentz transformation equations.

17. (*a*) A particle with a mean proper life time of 2 μsec moves through the laboratory with a speed of $0 \cdot 9c$. Calculate its life time as measured by an observer in laboratory. 3

(*b*) A rod of length 1 m, when the rod is in a satellite moving with velocity $0 \cdot 8c$ relative to laboratory, what is the length of the rod as determined by an observer (*i*) in the satellite and (*ii*) in the laboratory? 1+2=3

18. Establish the Einstein mass-energy relation, $E = mc^2$. 6

19. Calculate the velocity at which the mass of a particle becomes 8 times its rest mass. 2

20. Answer any *two* questions of the following : 6×2=12

(a) Find the transformation laws of momentum in relativistic mechanics. Can you compare the transformations with Lorentz transformations? 5+1=6

(b) Find the velocity that an electron must be given so that its momentum is 10 times its rest mass times the speed of light. What is the energy at this speed?

[Mass of the electron = 9×10^{-31} kg] 6

(c) A particle of rest mass m_1 and velocity v collides with a particle of mass m_2 at rest. After collision, the two particles become one. Show that the rest mass M and velocity V of the composite particle are given by

$$M^2 = m_1^2 + m_2^2 + \frac{2m_1 m_2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and $$V = \frac{m_1 v}{m_1 + m_2\sqrt{1 - \frac{v^2}{c^2}}}$$ 6

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