Total No. of Printed Pages—7

6 SEM TDC PHY M 1

2019

(May)

PHYSICS

(Major)

Course : 601

(Statistical Mechanics)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option of any *six* of the following : 1×6=6

(a) Macroscopic description of the state of a system corresponds to

(i) position co-ordinates of particles

(ii) momentum co-ordinates of particles

(iii) velocities of particles

(iv) pressure, volume, temperature, etc., of the system

(b) The dimension of the phase space of a particle moving in 2 dimensions is

(i) 2

(ii) 3

(iii) 4

(iv) 6

(c) If r_{av} represents average separation of particles and λ_{av} represents average de Broglie wavelength of particles, then for indistinguishable particles which one is correct?

(i) $r_{av} >> \lambda_{av}$

(ii) $r_{av} << \lambda_{av}$

(iii) $r_{av} \neq \lambda_{av}$

(iv) $r_{av} = 2\lambda_{av}$

(d) In grand canonical ensemble, systems

(i) can exchange energy only

(ii) can exchange particles only

(iii) can exchange both energy and particles

(iv) can't exchange energy and particles

(e) The statistics applicable to distinguishable (classical) particles is

(i) Maxwell-Boltzmann

(ii) Bose-Einstein

(iii) Fermi-Dirac

(iv) quantum statistics

(f) Pauli's exclusion principle is obeyed by

(i) spin-0 particles

(ii) spin-$\frac{1}{2}$ particles

(iii) spin-1 particles

(iv) spin-2 particles

(g) Electrons can be described by

(i) symmetric wave function

(ii) anti-symmetric wave function

(iii) both symmetric and anti-symmetric wave functions

(iv) None of the above

2. Answer any *six* from the following questions : $2 \times 6 = 12$

(a) Write down briefly the meaning of occupation number.

(b) Distinguish between micro-canonical and canonical ensembles.

(c) Write down the significance of occupation number.

(d) Define symmetric wave function and anti-symmetric wave function.

(e) Write down the relevant expressions of symmetric and anti-symmetric wave functions for a system of two particles.

(f) Define Fermi energy ε_F.

(g) A system can exist in three allowed states with energies ε_1, ε_2 and ε_3. Write down the partition function for the system. What is the probability that the system would be found in the state with energy ε_2 if a measurement is made?

3. Using Lagrange's method of undetermined multipliers, obtain the expression for occupation number for Maxwell-Boltzmann statistics. 6

4. (a) Based on the additive property of entropy, obtain the relation between entropy and probability. 3

(b) Obtain the relation between average energy and partition function. 3

(c) A particle has two allowed states with energies 0 and ε. Show that the partition function is given by

$$Z = 2e^{-\frac{\beta\varepsilon}{2}} \cosh\left(\frac{\beta\varepsilon}{2}\right) \quad \left[\beta = \frac{1}{kT}\right]$$ 3

5\. *(a)* A particle is confined in a volume υ. Show that the number of states available for the particle in its phase space is given by

$$\frac{2\pi\upsilon}{h^3}(2m)^{3/2}\int \varepsilon^{1/2}\, d\varepsilon$$

[Use the formula :

number of states $= \frac{1}{h^3}\int d^3r\, d^3p$] 6

Or

Derive the expression for Bose-Einstein distribution function. 6

(b) Write down Fermi-Dirac distribution function. What is its physical significance? Give your answer considering the situation at absolute zero temperature. 3

(c) At absolute zero temperature $(T = 0\,\text{K})$, all the energy levels up to ε_F are completely filled. Calculate the total number of fermions in a Fermi gas at $T = 0\,\text{K}$ and express ε_F in terms of number density $\left(N/V\right)$. 6

6. *(a)* Using Bose-Einstein distribution function, derive Planck's law of black-body radiation. 8

Or

Derive an expression for the pressure exerted by the Fermi gas in white dwarf stars. 8

(b) Explain qualitatively the meaning of Bose-Einstein condensation. 4

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