1 SEM TDC GEMT (CBCS)

GE 1 (A/B/C)

2021

(March)

MATHEMATICS

(Generic Elective)

The figures in the margin indicate full marks for the questions

Paper : GE–1 (A)

(Differential Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. *(a)* Define limit of a function. 1

(b) Choose the correct answer : 1

A function $f(x)$ is said to be continuous at $x = a$ if

(i) $\lim_{x \to a} f(x)$ exists

(ii) $f(a)$ exists

(iii) $\lim_{x \to a} f(x) = f(a)$

(iv) $\lim_{x \to a} f(x) \neq f(a)$

(c) Let f be the function given by

$$f(x) = \frac{x^2 - a^2}{x - a}, \; x \neq a$$

Using (ε, δ) definition, show that

$$\lim_{x \to a} f(x) = 2a$$ 2

(d) If f is even and differentiable function, prove that $f'(x)$ is odd. 2

2. (a) If $y = \cos 3x$, write y_n. 1

(b) If $y = x^{2n}$, where n is a positive integer, show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5 \cdot \cdots (2n-1)\} \, x^n$$ 3

(c) If $y = \cos(m \sin^{-1} x)$, show that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} + (m^2 - n^2) y_n = 0$$ 5

Or

If $f(x) = \tan x$ and n is a positive integer, prove with the help of Leibnitz's theorem that

$$f_{(0)}^{n} - {}^{n}C_2 \, f_{(0)}^{n-2} + {}^{n}C_4 \, f_{(0)}^{n-4} - \cdots = \sin \frac{n\pi}{2}$$

(d) Investigate the type of discontinuity of the function f at $x = 1$ defined by

$$\begin{aligned} f(x) &= 5x + 9, & x > 1 \\ &= 14x - 9, & x < 1 \\ &= 14, & x = 1 \end{aligned}$$ 4

Or

If $f(x)$ is continuous on an interval I, a and b are any two numbers of I, then show that if y_0 is a number between $f(a)$ and $f(b)$, there exists a number c between a and b such that $f(c) = y_0$.

3. *(a)* If

$$u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$$

find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$. 2

(b) If $y = f(x + ct) + \phi(x - ct)$, show that

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$ 4

Or

If $u = e^{xyz}$, prove that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$$

(c) State and prove Euler's theorem on homogeneous function of two variables. 5

Or

If

$$u = 2\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \cot\frac{u}{2} = 0$$

4. (a) Write the equation of normal to the curve $y = f(x)$ at the point (x_1, y_1). 1

(b) Define curvature of a curve at a point. 2

(c) Find the equation of the tangent to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where it meets x-axis. 3

Or

Find the curve whose curvature at any point on it is zero.

(d) With the help of a graph describe the motion of a particle whose position $P(x, y)$ at time t is given by parametric equations $x = a\cos t$, $y = b\sin t$; $0 \le t \le 2\pi$. 5

Or

Find the asymptotes of the graph of

$$f(x) = \frac{x^2 - 3}{2x - 4}$$

5. *(a)* Define double point of a curve. 1

(b) Find the Cartesian equation of the curve and identify the graph for the following polar equation : 3

$$r^2 = 4r\cos\theta$$

(c) Trace any *one* of the following curves : 5

(i) $y = x^3 - 12x - 16$

(ii) $r = 1 - \cos\theta$

6. *(a)* Choose the correct answer : 1

"If f is continuous in $[a, b]$, derivable in (a, b) and $f(a) = f(b)$, then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$." This result is known as

(i) Lagrange's theorem

(ii) Euler's theorem

(iii) Rolle's theorem

(iv) Taylor's theorem

(b) Write the remainder after n term of Taylor's theorem in Cauchy's form. 1

(c) Verify Rolle's theorem for the following function : 3

$$f(x) = \frac{x^3}{3} - 3x,\ x \in [-3, 3]$$

(d) If $f'(x) = g'(x)$ at each point of an interval I, then there exists a constant c. Prove that $f(x) = g(x) + c$, for all x in I. 3

(e) State and prove Lagrange's mean-value theorem. 5

Or

Using Maclaurin's series expand $\log(1+x)$ in an infinite series in powers of x.

7. *(a)* Define critical point of a function. 1

(b) Find the absolute extrema values of $f(x) = 8x - x^4$ on $[-2, 1]$. 4

(c) Evaluate (any *two*) : 3×2=6

(i) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

(ii) $\lim_{x \to \infty} \frac{x^4}{e^x}$

(iii) $\lim_{x \to 1} \left(\frac{1}{x^2 - 1} - \frac{2}{x^4 - 1} \right)$

(iv) $\lim_{x \to 0} (\sin x)^{2 \tan x}$

(d) State and prove Taylor's theorem with Lagrange's form of remainder. 6

Or

If f is continuous in $[a, b]$ and possesses first and second derivatives for $x = x_0$, where $a < x_0 < b$, prove that

$$f''(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

Paper : GE–1 (B)

(Object-oriented Programming in C++)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer any *ten* from the following questions : 1×10=10

   (a) What is a base class and a derived class?

   (b) How does a class enforce data hiding?

   (c) What is encapsulation?

   (d) Write the different access specifiers in a class.

   (e) What is an inline function?

   (f) What is a parameterized constructor?

   (g) Write the use of copy constructor.

   (h) Write the importance of destructor.

   (i) What is containership?

   (j) Write the use of array.

   (k) Enlist some advantages of OOP.

   (l) What is a function prototype?

2. Answer any *three* from the following questions : 2×3=6

(a) How are objects implemented in C++?

(b) Write the advantage of operator overloading.

(c) What is a reference variable? What is its usage?

(d) What is the difference between type casting and automatic type conversion?

(e) Write the output of the following code fragment :

```
int ch = 20;
cout << ch << ++ ch <<"\n";
```

3. Answer any *three* from the following questions : 4×3=12

(a) What is a member function? How does a member function differ from an ordinary function?

(b) Write four characteristics of a constructor function used in a class.

(c) What is a temporary instance of a class? What is its use and how is it created?

(d) Write down the various situations when a copy constructor is automatically invoked.

(e) How does the invocation of constructors differ in derivation of class and nesting of class?

4. Answer any *two* from the following questions : 7×2=14

(a) How does the visibility mode control the access of the members in the derived class? Explain with example.

(b) Explain different types of inheritances with example.

(c) Explain how the inheritance influence the working of constructor and destructor. Give example.

5. Answer any *three* from the following : 6×3=18

Write C++ program for the following :

(a) To demonstrate the use of constructor and destructor

(b) To reverse a given integer

(c) To demonstrate the access control in public derivation of a class

(d) To keep a count of created objects using static members

Paper : GE–1 (C)

(Finite Element Methods)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

1. *(a)* What is the concept of analysis in finite elements method? 1

   (b) Illustrate different types of finite elements and their uses in associated fields. 4

   Or

   Discuss briefly different types of partial differential equations used in the application of finite elements method.

   (c) Mention a particular initial boundary value problem and discuss it briefly in the field of finite element method. 4

   (d) Define a triangular finite element. Derive its formula in two-dimensional Cartesian region. 5

2. *(a)* Describe briefly about weighted residual methods. 3

   Or

   Find an expression for Galerkin collocation method.

(b) What do you mean by element assemblage in finite elements method? Give an example. 3

Or

Write down the importance of the application of finite elements method over certain practical domains.

3. (a) Find an expression for variational methods. 3

Or

Using Greens' theorem, find the Euler's equation.

(b) Write down the importance of shape functions in finite element methods. 4

Or

Discuss briefly about numerical methods used in solving partial differential equations.

(c) State and prove Dirichlet problem for Laplacian operator. 5

Or

Find the equation for weighted residual methods.

4. *(a)* What do you mean by conforming elements in finite element methods? 2

(b) Discuss about natural coordinates in the process of formulation of a linear Lagrange polynomial. 3

(c) Define a triangular element. Find an expression for linear Lagrange polynomial in case of triangular element. 4

(d) Find an interpolating function over an one-dimensional domain. 3

Or

Find a formulation for the solution of the boundary value problem ∇^2

$$u = 1, \quad |x| \le 1, |y| \le 1$$

$$u = 0,$$

on the boundary in case of triangular elements.

5. *(a)* Define weak derivatives with an example. 2

(b) Find an expression for rectangular elements and hence deduce its stiffness equation. 3

Or

Discuss about degrees of freedom of a finite element. Draw a picture of mixed plane elements with three degrees of freedom.

(c) Discuss about the importance of isoparametric elements in solving boundary value problems. 3

(d) Write down the procedure of calculating sparse matrix with the help of an example. 4

Or

Solve the boundary value problem

$$u'' + (1 + x^2)u + 1 = 0,\ u(\pm 1) = 0$$

with linear piecewise polynomial for two elements of equal length.

6. *(a)* Define Hilbert space. 1

(b) Find an expression for numerical integration over finite elements in one and two dimensions. 5

(c) Write a note on convergence analysis and completeness in Galerkin finite element methods. 6

Or

Find an algorithm for developing a formulation in line element mesh generation.

7. *(a)* Answer any *one* of the following : 4

(i) Find the assembly element equations of a line element while solving linear boundary value problem.

(ii) A triangular element (e) is expressed as linear function of x and y as follows :

$$u^{(e)}(x, y) = a + bx + cy$$

Find its shape function.

(b) Derive the local stiffness matrix for plane elasticity for a four-node rectangular element. 4

(c) Derive the boundary value problem that characterizes the minima of the functional

$$J[u] = \frac{1}{2}\int_0^1 [(u'')^2 - 2(u')^2 + u^2 - 2u]\, dx$$

$$u(0) = u'(0) = 0,\ u(1) = u'(1) = 0$$ 4

Or

Find the variational functional for the boundary value problem :

$$u'' = u - 4xe^x$$

$$u'(0) - u(0) = 1,\ u'(1) + u(1) = -e$$

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