

1 SEM TDC MTMH (CBCS) C1

2021

(March)

MATHEMATICS

(Core)

Paper : C-1

(**Calculus**)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Write the value of $\frac{d}{dt}(\tanht)$. 1
- (b) Write the value of $\frac{d^n}{dx^n}(\sin ax)$. 1
- (c) Write intervals in which $y = x^3$ is
concave up and concave down. 2

- (d) Determine the concavity of $y = 2 + \sin x$ on $[0, 2\pi]$. 2

- (e) Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$ 3

Or

$$\text{Show that } \cosh^{-1}\left(\frac{1}{x}\right) = \operatorname{sech}^{-1} x$$

- (f) Find y_n (any one) if—

(i) $y = \cos^3 x$;

(ii) $y = \frac{a-x}{a+x}$. 3

- (g) If $y = \tan^{-1} \frac{x}{a}$, then find y_n . 4

Or

If $\log y = \tan^{-1} x$, show that

$$(1+x^2)y_2 + (2x-1)y_1 = 0$$

- (h) Evaluate : 4

$$\lim_{x \rightarrow 0} \frac{x - \sin x \cos x}{x^3}$$

Or

Find the asymptote of the curve

$$y^3 - x^2y + 2y^2 + 4y + 1 = 0$$

2. (a) Evaluate : 2

$$\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$$

- (b) Evaluate (any one) : 4

(i) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx$

(ii) $\int \sec^6 x \, dx$

- (c) Obtain the reduction formula for

$$\int \tan^n x \, dx$$

4

Or

A region is enclosed by the triangles with vertices (0, 1), (1, 0), (1, 1). Find the volume of the solid generated by revolving the region about the y -axis.

- (d) The circle $x^2 + y^2 = a^2$ revolves round the x -axis. Find the volume so generated. 5

Or

A region bounded by the curve $y = \sqrt{x}$, the x -axis, and the line $x = 4$ is revolved about the x -axis to generate a solid. Find the volume of the solid.

3. (a) Write the parametric formula for $\frac{dy}{dx}$. 1
- (b) Write the equation of the circle in polar form. 1
- (c) Write the equivalent Cartesian equation of $r^2 \sin 2\theta = 2$. 2
- (d) The position of a particle moving in the xy -plane is given by $x = \sqrt{t}$, $y = t$. Find the path traced out by the particle. 2
- (e) Find a parametrization for the curve having the line segment with end points $(-1, -3)$ and $(4, 1)$. 4

Or

Parametric equations and parameter interval for the motion of a particle in xy -plane is given $x = \cos 2t$, $y = \sin 2t$, $0 \leq t \leq \pi$. Identify the particle's path by finding a Cartesian equation.

- (f) Find the length of the astroid
 $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$ 5

Or

Find the area of the surfaces generated by revolving the curve $x = \cos t$, $y = 2 + \sin t$, $0 \leq t \leq 2\pi$ about x -axis.

4. (a) Write the necessary condition for the vectors \vec{a} , \vec{b} , \vec{c} to be co-planar. 1

- (b) The position vector of a moving particle is given by

$$\vec{r} = \cos 2t \hat{i} + 2 \sin 4t \hat{j} + t^2 \hat{k}$$

Find the acceleration at any time t . 2

- (c) Find the volume of the parallelepiped whose edges are represented by

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k} \quad 3$$

Or

Let

$$\vec{R}(u) = 4\hat{i} + (u^2 + 6u^3)\hat{j} + u^2\hat{k}$$

Find $\int_1^3 \vec{R}(u) du$.

- (d) Evaluate $\vec{a} \times (\vec{b} \times \vec{c})$, where

$$\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{i} + 3\hat{j} - 2\hat{k} \quad 4$$

Or

Find the tangential component of acceleration of a moving particle.
