1 SEM TDC PHYH (CBCS) C 1

2021

(March)

PHYSICS

(Core)

Paper : C–1

(Mathematical Physics—I)

Full Marks : 53
Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×3=3

(a) The direction of $\vec{\nabla}\phi$ is always

(i) $\perp$ to the surface ϕ = constant

(ii) $||$ to the surface ϕ = constant

(iii) $\perp$ or $||$ depending upon the shape of surface

(iv) None of the above

(b) The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+3y=0$ are

(i) order 2 and degree 2

(ii) order 2 and degree 1

(iii) order 1 and degree 2

(iv) order 3 and degree 2

(c) $\vec{\nabla}\cdot\vec{A}$ behaves like

(i) a scalar quantity

(ii) a vector quantity

(iii) scalar or vector

(iv) None of the above

2. Prove that the function $f(x)$ given by

$$f(x)=|x-1|,\ x\in R$$

is not differentiable at $x=1$. 2

3. What are linear and non-linear ordinary differential equations? Give examples. 2

4. (a) Solve the following ordinary differential equations (any *one*) : 3

(i) $(x^2 + y^2)dy = xydx$

(ii) $\frac{dy}{dx} = \frac{x(y^2 + 1)}{(x + 1)}$

(b) Solve the following 1st order linear differential equations by using integration factor (any *two*) : 3×2=6

(i) $\frac{dy}{dx} + xy = x$

(ii) $\frac{dy}{dx} - \frac{y}{x} = 2x$

(iii) $x\frac{dy}{dx} + \frac{y}{x} = 3x$

5. Find the partial differentiation

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2} \text{ and } \frac{\partial^2 f}{\partial y^2}$$

for the following functions (any *one*) : 2

(i) $f(x, y) = x^4 - 2x^2y^2$

(ii) $f(x, y) = 2x^3 - x^2y^6$

6. Solve the following partial differential equations by the method of separation of variable (any *two*) : 2½+2½=5

(i) $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0, \quad u=u(x, y)$

(ii) $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u, \quad u=u(x, y)$

(iii) $\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}, \quad u=u(x, y)$

7. (a) What are scalar and vector fields? Give examples. 2

(b) Show that

$$\vec{\nabla}\left(\frac{1}{r}\right)=-\frac{\vec{r}}{r^3}$$

where $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$. 2

(c) Calculate

$$\vec{\nabla}\cdot\vec{r}$$

if

$$\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$$ 2

8. *(a)* What is the geometrical interpretation of gradient of a scalar function? 2

(b) Using Stokes' theorem, prove that

$$\oint \vec{r} \cdot d\vec{r} = 0$$

where

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$ 5

Or

Starting with Maxwell's equation, $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$, apply Gauss' theorem to show that

$$\int_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

(Symbols have their usual meanings.)

(c) Evaluate

$$\iint \vec{F} . \hat{n} dS$$

where

$$\vec{F} = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$$

S being the surface of the sphere having centre (3, − 1, 2) and radius 3. 5

9. (a) Define orthogonal curvilinear coordinates. 2

(b) What is the infinitesimal volume element in spherical polar coordinate system? Compute the volume of a sphere of radius R using the expression for infinitesimal volume element in spherical coordinates. 2+2=4

10. What is Poisson's distribution? Calculate mean of Poisson's distribution. 1+3=4

11. Compute the following integral : 2

$$\int_{-\infty}^{+\infty} \delta(2x-1) \cdot (1+x+x^2)\, dx$$

★★★