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3 SEM TDC MTMH (CBCS) C 7

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(Held in January/February, 2022)

MATHEMATICS

(Core)

Paper : C–7

(PDE and Systems of ODE)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Write the degree of the equation

$$x\left(\frac{\partial^2 z}{\partial x^2}\right)+\left(\frac{\partial z}{\partial y}\right)^2=\frac{\partial z}{\partial x}$$ 1

(b) Write Lagrange's subsidiary equation of

$$xzp+yzq=xy$$ 1

(c) Write the general solution of $pq=k$. 1

(d) Solve : 5

$$(y-zx)p+(x+yz)q=x^2+y^2$$

Or

Find the integral surface of $x^2p+y^2q+z^2=0$, which passes through the hyperbola $xy=x+y$, $z=1$.

(e) Show that the equations $xp-yq=x$ and $x^2p+q=xz$ are compatible. 5

2. (a) Write Charpit's auxiliary equations for $q = 3p^2$. 2

(b) Find complete integral of any *one* of the following : 4

(i) $q = (z + px)^2$

(ii) $q + px = p^2$

(iii) $z^2 = pqxy$

(c) Find a complete integral of

$$p_1^3 + p_2^2 + p_3 = 1$$ 6

Or

Solve the boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ with $u(0, y) = 8e^{-3y}$ by the method of separation of variables.

3. (a) Write the condition when the equation

$$Rs + Ss + Tt + f(x, y, z, p, q) = 0$$

is hyperbolic. 1

(b) Determine the nature of the equation

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$ 2

(c) Show that $u = f(x + y) + g(y - x)$ satisfies the equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

where f and g are functions. 2

(d) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form. 7

Or

Derive the one-dimensional heat equation.

4. (a) Write the general form of two-dimensional heat equation. 1

(b) Write one assumption on vibrating string problem. 1

(c) Solve

$$\frac{\partial^2 u}{\partial x^2} = k^2 \left(\frac{\partial u}{\partial t} \right)$$

when $u(0, t) = u(l, t) = 0$, $u(x, 0) = \sin \frac{\pi x}{l}$. 6

Or

Solve the two-dimensional heat equation by the method of separation of variables.

5. (a) Write the equation $3\frac{d^2 x}{dt^2} + 6\frac{dx}{dt} - x = t^2$ in normal form. 1

(b) Let $L \equiv D^2 + 2$, $f(t) = e^{2t}$, where $D \equiv \frac{d}{dt}$. Find $Lf(t)$. 2

(c) Transform the linear differential equation

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 2x = t^2$$

into a system of first-order differential equation. 2

(d) Describe Euler's method. 4

Or

Find the characteristic roots of the equation associated in the solution of

$$\frac{dx}{dt} = 3x + y, \frac{dy}{dt} = 4x + 3y$$

(e) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} - x - 3y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} + x = e^{3t}$$

6

Or

Find $y(0 \cdot 1)$, $y(0 \cdot 2)$ in the solution of $\frac{dy}{dx} = x + y$, $y(0) = 1$, by using Runge-Kutta method.

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