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3 SEM TDC STSH (CBCS) C 5

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–5

(Sampling Distribution)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×5=5

(a) "If $X_1, X_2, \cdots, X_n$ are independently and identically distributed random variables with $E(X_i) = \mu_1$, $V(X_i) = \sigma_1^2$, $(i = 1, 2, \cdots, n)$, then the sum $S_n = X_1 + X_2 + \cdots + X_n$ is asymptotically normal with mean $\mu = n\mu_1$ and variance $\sigma^2 = n\sigma_1^2$." This theorem is

(i) De-Moivre-Laplace theorem

(ii) central limit theorem (CLT) for i.i.d. variates

(iii) Lyapunov theorem

(iv) None of the above

(b) Area of the critical region depends on

(i) size of type–I error

(ii) size of type–II error

(iii) number of observations

(iv) value of the statistic

(c) The shape of chi-square distribution curve for χ^2 with d.f. 1 is

(i) a parabola

(ii) a hyperbola

(iii) an inverted J-shaped curve

(iv) a bell-shaped curve

(d) Mean of the F-distribution with degrees of freedom n_1 and n_2 for $n_2 > 2$ is

(i) $\frac{n_2}{n_1 - 2}$ (ii) $\frac{n_1}{n_2 - 2}$

(iii) $\frac{n_1}{n_1 - 2}$ (iv) $\frac{n_2}{n_2 - 2}$

(e) The degree of freedom for Student's t-distribution based on a random sample of size n is

(i) $n-1$

(ii) n

(iii) $n-2$

(iv) $\frac{n-1}{2}$

2. Answer the following in brief : 2×5=10

(a) What are the uses of order statistics?

(b) What do you understand by null hypothesis and alternative hypothesis? Explain with examples.

(c) Mention the null hypothesis to be tested and also the test statistics used in large sample test for single mean.

(d) What is chi-square variate?

(e) Describe briefly one application of F-distribution.

3. (a) Define the r-th order statistics $\chi_{(r)}$ and obtain its distribution function. 1+3=4

(b) State and prove Chebyshev's inequality. 1+4=5

Or

For geometric distribution $p(x) = 2^{-x}$; $x = 1, 2, 3, \cdots$, prove that Chebyschev's inequality gives $P[|X-2| \leq 2] > \frac{1}{2}$ while the actual probability is $\frac{15}{16}$. 5

4. *(a)* What are the required conditions for the validity of χ^2-test of goodness of fit between theory and experiment? 4

(b) Find the mode of χ^2-distribution with n d.f. 5

Or

If χ_1^2 and χ_2^2 are two independent χ^2-variates with n_1 and n_2 d.f. respectively, then show that $\frac{\chi_1^2}{\chi_2^2}$ is a

$$\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$$

variate. 5

5. Define Student's t-distribution and derive its probability distribution. 2+6=8

Or

Define F-statistic. Write down its probability density function. Describe the F-test for testing the significance of the equality of two population variances. 2+2+4=8

6. Describe in detail, how you will test the—

(a) difference of proportions for large sample;

(b) difference of standard deviations for large sample. 5+4=9

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