3 SEM TDC STSH (CBCS) C 7

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–7

(Mathematical Analysis)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) The set

$$S = \left\{\frac{1}{n},\ n \in N\right\}$$

is bounded, where (N is the set of natural numbers)

(i) the supremum 1 belongs to S and infimum 0 does not

(ii) the supremum n belongs to S and infimum 1 does not

(iii) the supremum 1 and infimum 0 both do not belong to S

(iv) None of the above

(b) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(c) The series ΣU_n of positive terms is convergent or divergent as

$$\lim_{n \to \infty} \frac{U_n}{U_{n+1}} > 1 \text{ or } < 1$$

then this test is known as

(i) comparison test

(ii) Raabe's test

(iii) D'Alembert's test

(iv) Cauchy's condensation test

(d) An alternating series $\Sigma(-1)^n a_n$, where $a_n \geq 0$ for all n, is convergent if

(i) $\{a_n\}$ is bounded

(ii) $\{a_n\}$ is convergent

(iii) $\{a_n\}$ is decreasing

(iv) $\{a_n\}$ is decreasing and $\lim a_n = 0$

(e) To which of the following, Rolle's theorem can be applied?

(i) $f(x) = \tan x$ in $[0, \pi]$

(ii) $f(x) = \cos\left(\frac{1}{x}\right)$ in $[-1, 1]$

(iii) $f(x) = x^2$ in $[2, 3]$

(iv) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

(f) Lagrange's form of remainder after n terms in Taylor's development of the function e^x in a finite form in the interval $[a, a+h]$ is

(i) $\frac{h^n}{n!}e^{a+\theta h}$

(ii) $\frac{h^n}{n!}e^{\theta h}$

(iii) $\frac{h^n(1-\theta)}{n!}e^{a+\theta h}$

(iv) $\frac{h^n(1+\theta)^n}{n!}e^{a+\theta h}$

(g) The n th divided difference of a polynomial of n th degree is

(i) always zero

(ii) always equal to n

(iii) always constant

(iv) not defined

(h) Which one of the following is not a transcendental equation?

(i) $2x - \log_{10} 2 = 7$

(ii) $5x - \log_{10} x = 7$

(iii) $e^x - 3x = 0$

(iv) $e^{-x} = \sin x$

2. Answer the following questions : 2×8=16

(a) Define limit point of a set. When is a set said to be closed?

(b) Define montone sequences.

(c) What is Cauchy's condensation test?

(d) Define Raabe's test.

(e) A function $f(x)$ is defined on R by

$$f(x) = x; \quad \text{if} \quad 0 \le x < 1$$
$$= 1; \quad \text{if} \quad x \ge 1$$

Does $f'(1)$ exist? Verify.

(f) State Rolle's theorem.

(g) Define operators E and Δ, and show that $E\Delta = \Delta E$.

(h) Explain the use of numerical integration in statistics.

3. Answer any *two* of the following questions : 7×2=14

(a) Define a bounded set and bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in N$, then show that

$$\underline{\lim}\left\{\frac{1}{a_n}\right\} = \frac{1}{\overline{\lim}_{a_n}}, \text{ if } \overline{\lim_{a_n}} > 0$$

2+5=7

(b) Prove that a set is closed iff its complement is open. Prove that the set of rational numbers in [0, 1] is countable. 4+3=7

(c) Define limit superior and limit inferior of a bounded sequence. Show that if (x_n) is a bounded sequence, then (x_n) converges iff $\lim\sup(x_n) = \lim\inf(x_n)$. 3+4=7

4. Answer any *two* of the following questions : 7×2=14

(a) Define D'Alembert's ratio test. By virtue of D'Alembert's ratio test, test whether the series

$$\sum \frac{n^2 - 1}{n^2 + 1} \cdot x^n, \; x > 0$$

is convergent or divergent. 3+4=7

(b) Define Leibnitz test for alternating series. Show that the series

$$\sum \frac{e^{-\lambda x} \lambda^x}{\underline{|x}}, \; \lambda > 0$$

convergent. 2+5=7

(c) State L'Hospital's rule. Evaluate

$$\lim_{x \to 0} \frac{x^2 - \sin^2 x}{x^4}$$

4+3=7

5. Answer any *two* of the following questions : 7×2=14

(a) Explain with example differentiability of functions. For what choice of a and b, the function

$$f(x) = \begin{cases} x^2, & x \le k \\ ax + b, & x > k \end{cases}$$

is differentiable at $x = k$? 3+4=7

(b) State Lagrange's mean value theorem and show that

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$$

if $0 < u < v$. 2+5=7

(c) State Cauchy's mean value theorem. Expand $\cos x$ in powers of x in infinite series using Maclaurin's series expansion. 2+5=7

6. Answer any *two* of the following questions : 7×2=14

(a) State Lagrange's interpolation formula and discuss its merits and demerits. Apply Lagrange's formula to find $f(5)$, given that $f(1) = 2$, $f(2) = 4$, $f(3) = 8$, $f(4) = 16$, $f(7) = 128$ and explain why the result differs from 2^5. 3+4=7

(b) State and prove Simpson's $\frac{1}{3}$rd rule for numerical integration. What is the effect of—

(i) change of origin;

(ii) change of scale on this rule? 5+2=7

(c) What is a polynomial? Define zero of a polynomial. Describe Newton-Raphson method. In which situation this method is applicable? 2+4+1=7

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