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3 SEM TDC PHYH (CBCS) C 5

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(Held in January/February, 2022)

PHYSICS

(Core)

Paper : C-5

(**Mathematical Physics—II**)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer : 1×5=5

(a) The value of $\operatorname{erf}_c(x) + \operatorname{erf}_c(-x)$ is

(i) 1

(ii) 2

(iii) -1

(iv) 0

(b) The value of $\Gamma\left(-\frac{1}{2}\right)$ is

(i) $\sqrt{\pi}$

(ii) $\frac{-\pi}{2}$

(iii) $-2\sqrt{\pi}$

(iv) 0

(c) The value of Legendre polynomial $P_2(x)$ is

(i) $(1-3x^2)$

(ii) $\frac{1}{2}(3x^2-1)$

(iii) $(3x^2-1)$

(iv) $\frac{1}{2}(1-3x^2)$

(d) The differential equation

$$\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2ny = 0$$

is known as

(i) Legendre's equation

(ii) Bessel's equation

(iii) Laguerre's equation

(iv) Hermite's equation

(e) The sum $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$ is

(i) $\frac{\pi^2}{12}$

(ii) $\frac{\pi^2}{6}$

(iii) $\frac{\pi^2}{8}$

(iv) None of the above

2. (a) State the Dirichlet's conditions for a Fourier series. 2

(b) Expand the function $f(x) = x \sin x$ in a Fourier series in the interval $-\pi \leq x \leq \pi$. Hence show that

$$\frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots = \frac{\pi - 2}{4} \quad 4+2=6$$

(c) Expand $f(x) = e^x$ in a cosine series over $(0, 1)$. 3

3. (a) What do you mean by ordinary and singular points of a differential equation? Find the nature of the point $x = -1$ with reference to the differential equation

$$x^2(x+1) \frac{d^2y}{dx^2} + (x^2 - 1) \frac{dy}{dx} + 2y = 0 \quad 1+2=3$$

(4)

(b) Solve the following using Frobenius method (any one) : 5

(i) $9x(1-x)y'' - 12y' + 4y = 0$

(ii) $xy'' + y + xy = 0$

(c) Express $2 - 3x + 4x^2$ in terms of Legendre polynomials. 3

Or

Prove that

$$\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$$

(d) Evaluate the following : 2+2=4

(i) $P_n(1)$

(ii) $\int_{-1}^1 P_3^2(x) dx$

4. Evaluate : 3

$$\int_0^{\infty} x^{n-1} e^{-h^2 x^2} dx$$

Or

Show that

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

5. Answer any two of the following : $3 \times 2 = 6$

(a) Find the absolute error, relative error and percentile error when 754126 is rounded to four significant digits.

(b) If $u = \frac{5x^3y^4}{z^5}$ and errors in x, y, z be 0.001, compute the relative maximum error when $x = y = z = 1$.

(c) State and prove the normal law of errors.

6. (a) Solve any two of the following partial differential equations by method of separation of variables : $4 \times 2 = 8$

$$(i) \frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

$$(ii) \frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x, \text{ under the conditions}$$

$$u = 0 \text{ at } t = 0; \frac{\partial u}{\partial t} = 0 \text{ at } x = 0$$

$$(iii) \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u, \text{ under the conditions}$$

$$\text{at } x = 0, u = 0 \text{ and } \frac{\partial u}{\partial x} = 1 + e^{-3y}$$

(6)

- (b) Find the solution of one-dimensional wave equation in Cartesian coordinates. 5

Or

Find the solution of 2-D Laplace's equation in cylindrical coordinates.
