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4 SEM TDC ECO M 1

2014

(May)

ECONOMICS

(Major)

Course : 401

(**Mathematics for Economics**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer : 1×8=8

(a) $A \cup A'$ is

(i) A

(ii) A'

(iii) Ω

(iv) ϕ

(b) Given $y = \log_{10} x$, $\frac{dy}{dx}$ is

(i) $\log_{10} x$

(ii) $\log_{10} e \times \frac{1}{x}$

(iii) $\frac{1}{x^2}$

(iv) $\frac{1}{x}$

(c) $\int_a^b f(x) dx = ?$

(i) $-\int_b^a f(x) dx$

(ii) $\int_b^a f(x) dx$

(iii) $\int_0^b f(x) dx$

(iv) $\int_a^0 f(x) dx$

(d) In the determinant $\begin{vmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix}$, the minor

of the element 8 is

(i) 0

(ii) 8

(iii) -3

(iv) -6

(e) Given the AR function $AR = 10 - 0.5q$, the MR function is

(i) $MR = -0.5q^2$

(ii) $MR = 10 - 0.5q^2$

(iii) $MR = 10 - q$

(iv) $MR = 10q - q$

(f) Rank of the matrix $\begin{bmatrix} 3 & 0 & 2 \\ -1 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$ is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(g) The function $f(x) = \frac{x^2 + 3x - 4}{x - 1}$ is not continuous at

(i) 1

(ii) 2

(iii) 3

(iv) None of the above

(h) $\int a^x dx = ?$

(i) $a^x + c$

(ii) $\log a^x + c$

(iii) $ax + c$

(iv) $\frac{a^x}{\log_e a} + c$

2. Answer any four of the following : $4 \times 4 = 16$

- (a) Find the numbers a and b that make A the inverse of B , when

$$A = \begin{bmatrix} 2 & -1 & -1 \\ a & \frac{1}{4} & b \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

- (b) Illustrate Hawkins-Simon conditions
 (c) Draw the graph of $xy = 1$
 (d) Derive the elasticity of substitution for Cobb-Douglas production function.
 (e) Evaluate :

$$\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 2}{x^2 + 5x - 6}$$

- (f) Given the input coefficient matrix

$$A = \begin{bmatrix} 0.05 & 0.25 & 0.34 \\ 0.33 & 0.10 & 0.12 \\ 0.19 & 0.38 & 0 \end{bmatrix}$$

Explain the economic meaning of the third column sum and the third row sum.

3. (a) (i) Define the following with examples :

$1 \times 4 = 4$

Null set ; Disjoint set ; Convex set ; Union of sets

- (ii) Define limit of a function.

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(iii) A function is given by

$$y = \frac{x^2 - 4x + 3}{x^2 + 2x - 3}$$

find whether the function is continuous at $x=1$ or not.

4

Or

(b) (i) If $A = \{2, 3, 4\}$, $B = \{2, 5, 6\}$; find

$$(A \cup B) \setminus (A \cap B)$$

3

(ii) Solve the following pair of equations graphically :

5

$$x + 4y = 2$$

$$6x + 8y = 24$$

(iii) Define continuity of a function.

3

4. (a) (i) Consider the following macro-economic model of two countries, $i = 1, 2$, that trade with each other :

$$Y_1 = C_1 + A_1 + X_1 - M_1, \quad C_1 = c_1 Y_1, \quad M_1 = m_1 Y_1$$

$$Y_2 = C_2 + A_2 + X_2 - M_2, \quad C_2 = c_2 Y_2, \quad M_2 = m_2 Y_2$$

Here $\forall i = 1, 2$; Y_i is income, C_i is consumption, A_i is (exogenous) autonomous expenditure, X_i denotes exports and M_i denotes imports of country i . Find the equilibrium values of Y_1 and Y_2 by matrix algebra.

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(ii) Distinguish between the following :

2+2=4

Static and Dynamic input-output models

Open and Closed input-output models

Or

(b) (i) Verify that the following matrix A is idempotent :

3

$$A = \begin{bmatrix} \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii) Given the technical coefficient matrix (A) and the final demand vector (F), find the consistent level of sectoral output in a static input-output framework :

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.3 \\ 0.3 & 0.5 & 0.2 \end{bmatrix} \quad F = \begin{bmatrix} 150 \\ 200 \\ 210 \end{bmatrix}$$

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5. (a) Distinguish between Cobb-Douglas production function and CES production function. State and prove the properties of CES production function.

2+10=12

Or

- (b) (i) A consumer has a utility function $u = u(x) = \alpha x^\beta$, $\alpha > 0$; $0 < \beta < 1$. Does the utility function display diminishing marginal utility? 5

- (ii) Find out $\frac{dy}{dx}$, when

$$y = \sqrt{\frac{1-x}{1+x}} \quad 3$$

- (iii) The AR function is given by $AR = 100 - 3q$. Find the elasticity of demand at $q = 5$. 4

6. (a) (i) Find $\int x \ln x dx$. 5

- (ii) Given the MC function

$$MC = Q^2 - 4Q + 3$$

find the level of output (Q) at which the AVC will be minimum. 6

Or

- (b) (i) Given the marginal propensity to import $M'(Y) = 0.1$ and the information that $M = 20$ when $Y = 0$, find the import function $M(Y)$. 4

- (ii) Define consumer's surplus. Given the demand function $p = 36 - q^2$ and the supply function $p = 6 + \frac{q^2}{4}$, find the consumer's surplus at equilibrium. 2+5=7

7. (a) (i) Let the demand and supply functions be

$$Q_d = \alpha - \beta P + \sigma \frac{dP}{dt}, \quad Q_s = -\gamma + \delta P$$

($\alpha, \beta, \gamma, \delta > 0$)

Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path $P(t)$. 7

- (ii) Briefly explain the use of differential equations in economics. 4

Or

- (b) (i) In a market model

$$Q_{dt} = 12 - 2P_t$$

$$Q_{st} = -4 + 2P_{t-1}$$

$$\text{and } P_{t+1} - P_t = -0.25(Q_{st} - Q_{dt})$$

Find the time path P_t and test whether the time path is convergent. 7

- (ii) Write a note on the cobweb model. 4
