5 SEM TDC GEMT (CBCS) GE 5 (A/B)

2021

(Held in January/February, 2022)

MATHEMATICS

(Generic Elective)

Paper : GE–5

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–5 (A)

(Mathematical Finance)

1. *(a)* Define cash flow stream. 1

   (b) Write True or False : 1

   Investments are described by assets.

   (c) If the interest rate is r, then write the price of an investment that pays P after one year. 2

   (d) Write about the investment and return for the situation represented by cash flow $(-2, 2 \cdot 2)$. 2

2. Answer any *two* from the following questions : 4×2=8

   (a) Explain the concept of comparison principle.

   (b) Explain risk aversion principle.

   (c) Describe the principle of compounding at various intervals.

3. Describe internal rate of return. 6

Or

Solve the equation $x^3 - x - 1 = 0$ by using bisection method.

4. *(a)* Write the difference between Short bond and Long bond. 1

   (b) Who does issue municipal bonds? 1

   (c) Define callable bond. 2

   (d) Define security. 2

5. Answer any *two* from the following questions : 7×2=14

   (a) Describe Macaulay duration.

   (b) State and prove the theorem of internal rate of return.

   (c) Describe term structure of interest rate.

6. *(a)* Define asset. 1

(b) Define rate of return. 1

(c) Explain shorting. 2

(d) Write reasons why short selling is risky. 2

7. Answer any *two* from the following questions : 7×2=14

(a) Define mean standard deviation diagram and show that

$$\text{var}(x) = E(x^2) - \bar{x}^2$$

(b) Explain random return.

(c) Describe portfolio mean and variance.

8. *(a)* Define risk-free asset. 1

(b) Write about betas of stocks. 1

(c) Define security market line. 2

(d) Write about Jensen's index. 2

9. Answer any *two* from the following questions : 7×2=14

(a) State and prove two-fund theorem.

(b) Describe one-fund theorem.

(c) Describe capital asset pricing model.

Paper : GE–5 (B)

(Queuing and Reliability Theory)

1. Answer any *four* from the following questions : 10×4=40

(a) Define queue. Obtain the average number of customers in the system and queue for M/M/1 queuing model. 3+3+4=10

(b) Write down the steady-state equations for M/M/1/K queuing model. Obtain the probability distribution of number of customers in the system. 5+5=10

(c) Let the random variable W_q, which denotes the waiting time in the queue of an arriving customer. Derive the probability density function of W_q. Assume that the queue discipline is FCFS. 10

(d) Suppose that customers arrive at a counter in accordance with a Poisson process with mean rate of 2 per minute. Then the interval between any two successive arrivals follows exponential distribution with mean ½ minute.

Obtain the probability that the interval between two successive arrivals is—

(i) more than one minute;

(ii) 4 minutes or less;

(iii) between 1 and 2 minutes. 3+3+4=10

(e) Define M/M/C queuing system. Draw the state transition rate diagram. For M/M/C queueing system, find—

(i) the expected number of customers in the queue;

(ii) the probability that an arriving customer has to wait in the queue.

5+5=10

(f) A company has a large number of machines. The number of machines, which need repairs in a day, has a Poisson distribution with mean 20. The company has three repairmen, who work in parallel. The required service time for any repairman is exponential with mean service rate of 20 machines per day. Assuming that a day is of 8-hour duration, find the following :

2×5=10

(i) How many hours of a day, all the 3 repairmen are busy?

(ii) What is the probability that all the 3 repairmen are idle, 2 are idle and one is idle?

(iii) Find average waiting time in system and queue and average number of customers in the system.

(iv) Find the probability that an arriving customer has to wait for service.

(v) If the number of repairmen is increased by one, what is the probability that an arriving customer will go straight into service?

2. Answer any *three* from the following questions : 10×3=30

(a) What is reliability? Why is it important to study? Discuss the use of reliability in day-to-day life. 3+3+4=10

(b) Define the following : 2×5=10

(i) Mean time to failure (MTTF)

(ii) Mean time to repair (MTTR)

(iii) Mean time between failures (MTBF)

(iv) Probability of failure on demand (POFOD)

(v) Rate of occurrences of failure (ROCOF)

(c) Consider a system consisting of n components such that the failure of the ith component occurs in accordance with a Poisson process of intensity a_i. Find the reliability of the system under—

(i) series system;

(ii) parallel system;

(iii) k-out of n-system. 3+4+3=10

(d) Find the reliability of a system consisting of n identical components connected in parallel, such that the failure of a component occurs independently of the others and in accordance with a Poisson process with intensity a. 10

3. Write short notes on any *two* of the following : 5×2=10

(a) Preventive and corrective maintenance

(b) Performance measures of queuing system

(c) Single-server and multi-server queueing models with examples

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