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2 SEM TDC MTH M 1

2014

(May)

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Matrices

(*Marks* : 20)

1. *(a)* State True or False : 1

If A is a non-zero matrix, then rank $(A) < 1$.

(b) Define elementary matrix. Also find the rank of the matrix $E_{12}(3)$ considering I_3. 2

(c) Find the rank of the following matrix reducing it into normal form : 5

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

Or

Reduce the following matrix A to Echelon form and hence find its rank :

$$A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$$

2. (a) Write down the condition under which the system of equations $AX = B$ possesses a unique solution. 1

(b) Show that a characteristic vector of a matrix cannot correspond to more than one characteristic value of A. 2

(c) Show that the only real value of λ for which the following equations have non-zero solution is 6 : 3

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x + 3y + z = \lambda z$$

(d) Show that the following system of equations is consistent and solve them completely : 2+4=6

$$x+y+z=6$$
$$x+2y+3z=14$$
$$x+4y+7z=30$$

Or

State Cayley-Hamilton theorem. Show that the matrix

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

satisfies Cayley-Hamilton theorem. 1+5=6

(B) Ordinary Differential Equations

(*Marks* : 30)

3. (a) Write True or False : 1

"The singular solution of a differential equation in Clairaut's form contains only one arbitrary constant."

(b) Find the integrating factor of the differential equation

$$(x^3+y^3)dx-xy^2dy=0$$ 2

(c) Solve any *one* : 3

(i) $y = (1 + p)x + ap^2$, where $p = \dfrac{dy}{dx}$

(ii) $\dfrac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

(d) Use Wronskian to show that the functions x, x^2, x^3 are linearly independent. Determine the differential equation with these as independent solutions. 4

Or

Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine this differential equation.

4. (a) What is the auxiliary equation of the differential equation

$$\frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0$$

where P_1 and P_2 are constant? 1

(b) Define linear homogeneous equation. 1

(c) Solve any *two* : 4×2=8

(i) $(D+3)^2 y = 0$

(ii) $\frac{d^2y}{dx^2} = \cos nx$

(iii) $\frac{d^2y}{dx^2} - y = xe^x \sin x$

(d) Solve any *two* : 5×2=10

(i) $\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = e^x \sec x$

(By removing 1st order derivative)

(ii) $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

(By changing the independent variable)

(iii) $\frac{d^2y}{dx^2} + x^2 y = \sec nx$

(By the method of variation of parameters)

(C) Numerical Analysis

(*Marks* : 30)

5. (*a*) State True or False : 1

The bisection method always converges.

(*b*) Write the basic difference between the bisection method and method of false position. 1

(*c*) Explain the geometrical interpretation of the Newton-Raphson method for solving an algebraic equation. 3

(*d*) Answer any *two* : 5×2=10

(*i*) Describe the regula-falsi method for obtaining a real root of an algebraic equation.

(*ii*) By using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$, which is nearer to $x = 2$, correct to three decimal places by performing at least 3 iteratives.

(*iii*) Solve the following equations by Gauss elimination method :

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

6. (a) State True or False : 1

Simpson's one-third rule is better than the trapezoidal rule.

(b) Evaluate $\Delta^3(e^{ax+b})$ the interval of differencing being h. 2

(c) Show that $E \equiv 1 + \Delta$, where the symbols have their usual meanings. 2

(d) Answer any *two* : 5×2=10

(i) Deduce Lagrange interpolation formula.

(ii) Estimate the missing term in the following table :

x	:	0	1	2	3	4
$f(x)$	:	1	3	9	?	81

(iii) Show that $\int_0^1 \frac{dx}{1+x} = \log 2 = 0 \cdot 69315$, by dividing the range into 10 equal parts.

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